Endogenous Uncertainty and Credit Crunches*

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Abstract

We develop a theory of endogenous uncertainty in which the ability of investors to learn about firm-level fundamentals is impaired during financial crises. At the same time, higher uncertainty reinforces financial distress. Through this two-way feedback loop, a temporary financial shock can cause a persistent reduction in risky lending, output, and employment that coincides with increased uncertainty, default rates, credit spreads and disagreement among forecasters. We embed our mechanism into standard real business cycle and New-Keynesian models and show how it generates endogenous and internally persistent processes for the efficiency and labor wedges.

Keywords: Endogenous uncertainty, financial crises, internal persistence.
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1 Introduction

Financial crises often entail deep and long-lasting recessions (Reinhart and Rogoff, 2009; Hall, 2014; Ball, 2014). A common view gives a central role to uncertainty, both as an amplifier of financial distress and a source of slow recovery.[1] This paper explores this idea, developing a theory that formalizes the interaction between financial constraints and uncertainty.

Our theory provides a narrative of how a temporary shock emanates from the financial sector, is reinforced by endogenously rising uncertainty, and ultimately develops into a long-lasting crisis of the real economy. The theory is consistent with a number of stylized facts from previous financial crises, such as the one in 2008/09: (i) persistently depressed employment and output; (ii) large credit spreads; (iii) a rise in default rates; (iv) an increased cross-sectional dispersion of firm sales; (v) the contemporaneous increase in measured uncertainty; and (vi) high levels of disagreement among forecasters.[2]

At the core of our theory is a two-way interaction between firms' access to external funds and information. Firms require bank loans to operate their businesses at potential. The ability of a firm to obtain funding hinges on how banks perceive the quality of its potential. The more pessimistic or uncertain banks are about the firm's potential, the less likely the firm obtains funding. Vice versa, when a firm is unable to operate at potential, less information about its quality is being generated, increasing uncertainty. Jointly, these two forces imply that an exogenous, but temporary, reduction in funding can morph into a persistent spiral of increased uncertainty about a firm's potential, heightened credit spreads, and the firm operating below its potential.

We embed this mechanism in a neoclassical general equilibrium model with a representative household and heterogeneous firms, which are funded by a competitive banking sector. We show that the amplification and internal persistence inherent in the mechanism carries over to aggregate financial shocks that hit banks' capacity to lend. Calibrating our model to U.S. data, we simulate an aggregate financial shock with a half-life of 4 quarters. We find that

[1] For example, Olivier Blanchard (2009) speculated at the height of the recent financial crisis that "the crisis would largely go away" if it were not for uncertainty, whereas Bloom et al. (2018) document how uncertainty was repeatedly recognized by the Federal Open Market Committee as a driver of both, the recession that followed the dot-com bubble in 2001, and the recent Great Recession. An increasing number of empirical studies further substantiates these ideas, pointing to the Great Recession being likely "an acute manifestation of the toxic interaction between uncertainty and financial shocks" (Caldara et al., 2016; see also Stein and Stone, 2013, Stock and Watson, 2012, and Gilchrist, Sim and Zakrajšek, 2016).

[2] The rise in credit spreads during the 2008/09 financial crisis has been documented in, e.g., Gilchrist and Zakrajšek (2012); the rise in default rates has been documented in, e.g., Gourio (2014); the rise in sales dispersion has been documented by Bloom et al. (2018); unusually high levels of uncertainty have been documented using a variety of different approaches, including Jurado, Ludvigson and Ng (2015), Born, Breuer and Elstner (2018), and the studies cited in Footnote 1; and the increase in disagreement has been documented by Senga (2018).

the persistence of the output response in our model is much greater than that, with a half-life of 16 quarters. The discrepancy is caused entirely by the interaction between endogenous uncertainty and financial frictions: when shutting down the former, the half life of the output response falls to 4 quarters, mirroring the half-life of the exogenous financial shock.

For illustrative purposes, our baseline model is stylized and does not feature capital. Nevertheless, as we demonstrate in three extensions, it is straightforward to incorporate our mechanism into richer environments. First, we explore a variant of our model, in which a fraction of firms does not rely on external funds to finance their projects. While the presence of such firms scales down the overall impact of financial shocks, we find that it changes little about their propagation through endogenous uncertainty and does not reduce the internal persistence.

Second, we extend our baseline model to include investment and capital. Interestingly, we show that our model—with its firm-level heterogeneity and two-way interaction between lending and beliefs about firm potential—is observationally equivalent to a standard real business cycle (RBC) model with endogenous processes for the economy's "efficiency wedge" and "resource wedge", in the spirit of Chari, Kehoe and McGrattan (2007). These wedges arising from our mechanism are different from the ones in existing models based on financial frictions such as Buera and Moll (2015) in their internal persistence after a financial shock.

Third, we develop a New Keynesian version of our model, with nominal rigidities and hand-to-mouth households, following Galí, López-Salido and Vallés (2007) and Bilbiie (2008). We show that in this extension, as well, financial shocks lead to a protracted decline in output due to endogenous uncertainty. However, in contrast to our baseline model, the propagation now runs through the demand side, driven by a reduction in household income and consumer spending, manifesting itself as a persistent increase in the economy's labor wedge.

While the aggregate dynamics of the model are fully captured by endogenous wedges, our model also has implications at the firm level. In particular, as mentioned above, rising uncertainty helps explain a variety of financial market characteristics associated with financial crises: increased credit spreads, a rise in default rates, an increased cross-sectional dispersion of firm sales, and high levels of disagreement among forecasters about firm-level profitability.

To gauge the quantitative potential of our endogenous uncertainty mechanism, we estimate the RBC version of our model to historical data on U.S. business cycles, allowing for three typical business cycle shocks and the financial shock. We find that typical recessions, driven by the standard business cycle shocks, look similar with and without endogenous uncertainty. Recessions partly caused by financial shocks, however, are significantly more severe in the economy with endogenous uncertainty compared to an exogenous uncertainty counterfactual. In case of the Great Recession, we find that without endogenous uncertainty, the peak-totrough drop in output would have been about half of what it was, and output would have fully recovered by 2010.

All uncertainty in our model is about firm-level fundamentals, not aggregate fundamentals. In the final section of our paper, we show that this matters: since aggregate shocks are much smaller than firm-level ones, we do not find a large role for endogenous *aggregate* uncertainty in response to financial shocks.

Related literature. Our paper is related to a large and growing literature that introduces dispersed information into macroeconomics (e.g., Lorenzoni, 2009; Angeletos and La'O, 2010, 2013; Amador and Weill, 2010, 2012; Maćkowiak and Wiederholt, 2015; Hassan and Mertens, 2014, 2017; Acharya, 2013; Hellwig and Venkateswaran, 2014; Chahrour and Gaballo, 2021). La'O (2010) shares with us the combination of dispersed information with financial frictions, but considers a static model with a constant level of uncertainty. David, Hopenhayn and Venkateswaran (2016) also analyze information frictions as a source for factor misallocation, but focus on long-run consequences rather than fluctuations driven by financial shocks.

Our paper also contributes to a recent literature that explores the role of endogenous fluctuations in uncertainty for business cycles, including van Nieuwerburgh and Veldkamp (2006), Ordoñez (2013), and Fajgelbaum, Schaal and Taschereau-Dumouchel (2017).[3] In these papers, the level of aggregate investment determines the amount of information and hence aggregate uncertainty. An important distinction relative to these papers is this paper's focus on uncertainty regarding firm-specific fundamentals rather than economic aggregates (see Senga 2018 for a similar approach). On the one hand, this allows us to explain the aforementioned stylized facts regarding the cross-sectional distribution of firm sales and investor beliefs. On the other hand, this also helps overcome a challenge in the endogenous uncertainty literature; namely, that it is often hard to generate meaningful endogenous fluctuations in uncertainty. In our model, by contrast, learning slows down when *a firm* is unable to obtain funding to operate at its potential, not when aggregate economic activity comes to a stand-still. Thus, even small aggregate perturbations can get severely amplified. A second important difference to the existing endogenous uncertainty literature is that this paper links *financial* crises and uncertainty through a novel mechanism, explaining why high

[3] Ilut and Saijo (2021) propose a related mechanism based on ambiguity aversion. Studies of endogenous uncertainty in financial market settings include Veldkamp (2005), Yuan (2005), Albagli (2011), and Sockin (2017). However, none of these papers considers spillovers from financial distress on the real economy that are at the core of this paper. Benhabib, Liu and Wang (2019) and Gaballo and Marimon (2021) study settings, in which an informational interdependence between financial markets and learning gives rise to self-fulfilling fluctuations. Finally, Gorton and Ordoñez (2014) and Asriyan, Laeven and Martin (2021) study the reverse scenario in which a depletion of information makes the economy prone to fall into credit crises.

levels of uncertainty are particularly prevalent during financial crises.[4]

In our model, the emergence of uncertainty due to financial distress interacts with the propagation of uncertainty through the financial sector. In support of such a financial transmission channel, Gilchrist, Sim and Zakrajšek (2016) present evidence that uncertainty strongly affects investment *via* increasing credit spreads, but has virtually no impact on investment when controlling for credit spreads. The financial transmission of uncertainty relates our model to a recent literature around Christiano, Motto and Rostagno (2014), Arellano, Bai and Kehoe (2019), and Gilchrist, Sim and Zakrajšek (2016), which stresses the importance of uncertainty or risk shocks in the financial sector, but treats these shocks as exogenous.[5]

The predictions of our model are also broadly consistent with a recent empirical literature that measures the effects of tightening financial constraints. Giroud and Mueller (2017) show that establishments of firms that are more likely to be financially constrained were heavily affected by falling collateral values (house prices). In fact, they show that the entire correlation of employment loss and house prices is explained by these arguably financially constrained firms. Similar in spirit, Chodorow-Reich (2013) and Huber (2018) document that firms borrowing from less healthy lenders experience relatively steeper declines in employment during the financial crisis, consistent with the interpretation that these firms faced tighter financial constraints. Our model clarifies how an intense but relatively short-lived financial crisis can still translate into persistent financial constraints for firms, making it much harder for them to weather such periods and retain their employment and capital.

Outline The plan for the rest of the paper is as follows. The next section introduces the model economy and characterizes equilibrium. Section 3 explores our main mechanism focusing on the partial equilibrium dynamics of a single firm. Section 4 analyzes the general equilibrium response to an aggregate financial shock. Section 5 explores the models with capital and with nominal rigidities, and introduces heterogeneity in firms' reliance on external funds. Section 6 provides a quantitative exploration of the model. Section 7 studies a variant of our model, in which uncertainty is about aggregate productivity. Section 8 concludes. Appendix A presents evidence from survey data in support of the the main mechanism.

[4]We show in Appendix D.1 that our mechanism does *not* cause meaningful increases in uncertainty after exogenous shocks to aggregate productivity. This is consistent with the evidence in Figure A.I.

[5]Two other related strands of the literature study the propagation of exogenous uncertainty through real options as in Bloom (2009), Bloom et al. (2018), and Bachmann and Bayer (2013), and through risk premia as in the time-varying (disaster) risk literature (e.g., Gabaix, 2012; Gourio, 2012). Related to the latter, Kozlowski, Veldkamp and Venkateswaran (2020) explore a model where agents learn about tail-risks and where belief revisions after short-lived financial shocks can have long-lasting effects. Similar, Nimark (2014) presents a mechanism that increases uncertainty after rare events, if news selectively focus on outliers.

2 Baseline Model

We study our mechanism in a neoclassical economy with a representative household, a competitive final goods sector, and a continuum of monopolistically competitive intermediate-goods firms. The latter are partially funded by a competitive banking sector. Time is discrete with an infinite horizon and is indexed by t. To illustrate the mechanism, our baseline model abstracts from capital, nominal rigidity and non-credit based funding. We study the consequences of adding those features to our model in Sections 5 and 6.

2.1 Environment

Firms. A competitive final-good sector combines intermediate goods, $\{Y_{i,t}\}_{i\in[0,1]}$, to produce final output, Y_t, using the technology

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\xi-1}{\xi}}\,\mathrm{d}i\right)^{\frac{\xi}{\xi-1}},$$

where $\xi > 1$ is the elasticity of substitution between input varieties. Profit maximization yields the demand for input i with price $p_{i,t}$,

$$Y_{i,t} = Y_t p_{i,t}^{-\xi}, \tag{1}$$

where the aggregate price index $P_t = \left(\int_0^1 p_{i,t}^{1-\xi}\,\mathrm{d}i\right)^{1/(1-\xi)}$ has been normalized to 1.

Each input, $i \in [0,1]$, is produced by a monopolistically competitive firm that operates a linear production technology,

$$Y_{i,t} = \mathcal{A}_{i,t}^{\frac{1}{\xi-1}} L_{i,t}, \tag{2}$$

where $L_{i,t}$ are units of labor. Here, the exponent on $\mathcal{A}_{i,t}$ is chosen so that $\mathcal{A}_{i,t}$ corresponds to firm i's revenue productivity, which simplifies the exposition below. In any given period t, $\mathcal{A}_{i,t}$ takes one of two values, $\{A_{i,t}, \tilde{A}\}$. We refer to $A_{i,t}$ as the productivity of the *risky* technology (or risky productivity), and to $\tilde{A}$ as the productivity of the *baseline* technology (or baseline productivity). We interpret the risky technology as capturing a firm's potential.

While the baseline technology has a constant productivity $\tilde{A} > 0$, the log productivity of the risky technology, $\log A_{i,t}$, evolves according to an AR(1) process,

$$\log A_{i,t} = \rho \log A_{i,t-1} + (1-\rho)\log \bar{A} + \epsilon_{i,t}, \tag{3}$$

with persistence $\rho \in (0,1)$, a long-run mean $\log \bar{A}$, and i.i.d. (across firms and time) Gaussian innovations $\epsilon_{i,t}$ with zero mean and variance σ_ϵ^2.

Conditional on period-t productivities and given a real wage w_t, firms choose $p_{i,t}$ to maximize operating profits,

$$\Pi_{i,t} \equiv p_{i,t} Y_{i,t} - w_t L_{i,t} \tag{4}$$

subject to (1) and (2).

Which firm produces using which of the two productivity levels is determined by two interacting frictions: a financial friction and an informational friction. We explain them next, beginning with the financial friction.

Financial friction. Each period has two sub-periods, a morning and an afternoon.

In the morning, firms choose whether to operate the baseline technology, with productivity $\tilde{A}$, or the risky technology, with productivity $A_{i,t}$. Operating the baseline technology entails an upfront operating cost of $\tilde{\phi} > 0$, whereas operating the risky technology entails a larger upfront cost of $\phi > \tilde{\phi} > 0$. Importantly, the technology choice is made subject to an information set $\mathcal{I}_t$ (detailed below), which does not contain the current realization of $A_{i,t}$. This is why the "risky technology" is indeed risky. Conditional on their technology choice, firms then approach banks to finance the upfront cost $\phi_{i,t} \in \{\phi, \tilde{\phi}\}$.

In the afternoon, firms produce, goods are sold, wages are being paid, loans are repaid, and the household consumes.

We assume that a liquidity constraint prevents firms from using their afternoon profits to pay for the upfront cost $\phi_{i,t}$. Instead, firms borrow from a competitive banking sector in the morning, at an interest rate $r_{i,t}$, and repay their loans in the afternoon. When a firm is unable to do so due to its operating profits falling short of the repayment,

$$\Pi_{i,t} < (1 + r_{i,t})\phi_{i,t}, \tag{5}$$

it defaults on its loan. We assume that in case of default, banks need to pay a cost verifying the firms' default à la Townsend (1979), amounting to the firm's profits $\Pi_{i,t}$.[6] For simplicity, we assume that these costs are not resource costs and instead transfer from banks to households. If a firm defaults, it gets a bankruptcy flag that precludes it from obtaining risky loans, and thus precludes it from operating the risky technology. At the beginning of each period, bankruptcy flags are removed with an exogenous recovery probability $\eta \in (0, 1]$.

The interest rate $r_{i,t}$ compensates banks for the default risk. It is determined as the

[6] Costly state verification can also be used to show that debt contracts are optimal in this setting. Absent the verification cost the Modigliani–Miller theorem would apply and lending would be frictionless.

solution to the zero profit condition[7]

$$(1 + r_{i,t}) \left(1 - \mathbb{P}_t\big(\Pi_{i,t} < (1 + r_{i,t})\phi_{i,t}\big)\right) = 1 + \lambda_t. \tag{6}$$

The left-hand side of (6) corresponds to the expected return on lending one unit: $(1 + r_{i,t})$ is the return if the loan is repaid, and $1 - \mathbb{P}_t\big(\Pi_{i,t} < (1+r_{i,t})\phi_{i,t}\big)$ is the probability of repayment. The right-hand side is the cost of funds for banks and is subject to an exogenous financial shock $\lambda_t > -1$, which will be the source of aggregate credit crunches in the model. We assume $\lambda_t = 0$ in steady state, corresponding to a zero cost of funds for within-period lending. When λ_t rises above 0, this indicates that banks face an increased shadow cost of lending and are therefore required to raise lending rates. As such, our financial shock is similar to shocks to intermediary net worth, as in Gertler and Karadi (2011) and Gertler and Kiyotaki (2011).

An immediate implication of the zero profit condition (6) is that when the productivity $\mathcal{A}_{i,t}$ is known in advance, which is the case for firms operating the baseline technology, the interest rate equals the banks' cost of funds; that is, $r_{i,t} = \lambda_t$.[8] For firms operating the risky technology, there is a positive default premium $1 + r^p_{i,t} \equiv (1 + r_{i,t})/(1 + \lambda_t) > 1$. When $\lambda_t \neq 0$, the banking sector transfers its surplus $T^{\text{banks}}_t = \lambda_t \int_0^1 \phi_{i,t}\, \mathrm{d}i$ to the representative household.

Representative household. The representative household maximizes expected utility over consumption C_t and labor L_t,

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\log C_t - \upsilon \frac{1}{1+\zeta^{-1}} L_t^{1+\zeta^{-1}}\right), \tag{7}$$

where $\beta \in (0, 1)$ is the discount factor, $\upsilon > 0$ is a scale parameter, and $\zeta > 0$ is the Frisch elasticity of labor supply. The flow budget constraint is given by

$$C_t + B_{t+1} = w_t L_t + (1 + r_{t-1})B_t + T_t. \tag{8}$$

Here, B_{t+1} is the household's end-of-period t holding of a risk-less asset in zero net supply, w_t is the real wage, and r_t is the real interest rate. T_t represents several lump-sum payments: non-defaulting firms' operating profits $\Pi_{i,t}$, defaulting firms' bankruptcy transfer $\Pi_{i,t}$, and

[7] In case there are multiple solutions, $r_{i,t}$ is given by the smallest one. In case there are no solutions, a firm is unable to operate the risky technology and is forced to choose the baseline technology (which always permits a solution to (6) as long as operating is profitable).

[8] Here we tacitly assume that operating the baseline technology is profitable, which we impose more formally below.

banks' surplus T_t^{banks}. Taken together, T_t can be written as

$$T_t = \int_0^1 (\Pi_{i,t} - \phi_{i,t})\,\mathrm{d}i.$$

Information friction. We consider a simple information structure where all learning is public and there is no aggregate uncertainty; i.e., agents have complete information about the history of λ_t and the *shape* of the cross-sectional distribution over $A_{i,t}$. The only source of uncertainty is a lack of information about the productivities of the risky technology of each *individual* firm. Specifically, each period, after the technology adoption choice and before firms set prices, all agents observe the realized risky productivities for all firms adopting the risky technology. By contrast, for firms adopting the baseline technology, current risky productivities are only observed with an exogenous probability $\theta \in [0, 1)$, independently across firms, and remain otherwise unknown. Let $\mathcal{B}_t$ denote the set of firms that either adopt the risky technology in period t or for which $A_{i,t}$ is exogenously revealed. Then the information available to agents in the morning of date t is

$$\mathcal{I}_t = \lambda_t \cup \{A_{i,t-1}\}_{i \in \mathcal{B}_{t-1}} \cup \mathcal{I}_{t-1}.$$

These assumptions imply that the common belief entertained about each firm's risky productivity is log-normal at all times, allowing us to track the public beliefs in terms of each firms' expected log-productivity and the corresponding uncertainty,

$$\mu_{i,t} \equiv \mathbb{E}_t[\log A_{i,t}|\mathcal{I}_t] \qquad \Sigma_{i,t} \equiv \mathrm{Var}_t[\log A_{i,t}|\mathcal{I}_t].$$

Timing and market clearing. The timing of events within each period can be summarized as follows:

- *Morning:* Bankruptcy flags are removed with probability η; firms choose their technology; firms approach banks for funding and pay the operating cost $\phi_{i,t}$.

- *Afternoon:* Risky productivities $A_{i,t}$ are revealed for all firms operating the risky technology and with probability θ for all other firms; firms hire labor, produce, set prices, and repay loans; if firms are unable to repay, they default and get a bankruptcy flag; dividends and transfers are paid; the household consumes.

In equilibrium, the representative household chooses C_t, L_t and B_t to maximize utility (7), firms choose their technology and set prices to maximize profits, banks lend if their zero profit condition can be satisfied at the competitive default premium, and markets clear: labor markets satisfy $\int_0^1 L_{i,t}\,\mathrm{d}i = L_t$, goods markets satisfy

$$Y_t = C_t + \int_0^1 \phi_{i,t}\,\mathrm{d}i, \tag{9}$$

and asset markets satisfy $B_t = 0$ at all times t.

Below, we work with a parameterization of the model in which firms using the baseline technology always make positive profits; and in which firms that can get a bank loan for the risky technology always find it optimal to do so.[9] The former assumption ensures that firms prefer operating the baseline technology to exiting; the latter assumption ensures that the financial friction has an impact on firm behavior.

Discussion. Two ingredients are at the core of our model. First, firms rely, at least in part, on external finance, and access to external finance hinges on the perceived quality and risk of their production potential. We model this by assuming that there is an upfront operating cost that needs to be financed through loans. In this environment, more pessimistic and/or uncertain beliefs by financial markets naturally reduce access to loans, because they translate into greater default risk, raising credit spreads.[10] In our baseline model all firms have ex-ante the same reliance on external finance. Ex-post, the ones that are perceived as more productive have no issues securing funding at costs close to the internal bank rate λ_t. In Section 5.3, we demonstrate that our mechanism is robust to also allowing for ex-ante heterogeneity in reliance on external funding. We do so by letting some firms fund the operating cost frictionlessly (e.g., due to equity, retained earnings or available safe collateral).

Second, a lack of funding leads to a lack of information about firms' potential productivity. In our model, firms that do not operate the risky technology generate less information about its productivity $A_{i,t}$. In reality, the risky technology captures a firm's potential, which is ex-ante uncertain. The longer a firm remains underfunded, unable to reach and test its potential, the less clear it becomes how profitable it actually is. Observe that $A_{i,t}$ need not correspond to productivity in reality. It could equally well capture firm-specific demand shifters; the two are isomorphic from a modeling perspective.

Finally, while we formalize the impact of being constrained in terms of firm productivity, one may equivalently think of it in terms of variations in factor utilization or differences in returns across a firm's projects. When we calibrate the model in Section 4.1, we will

[9]We can state the former assumption formally as $\tilde{A}\xi^{-\xi}(\xi-1)^{\xi-1}\,Y_t/w_t^{\xi-1} > (1+\lambda_t)\,\tilde{\phi}$. The latter assumption is more complex as firms internalize how uncertainty affects future access to credit and profits. We verify that it holds numerically in our calibration.

[10]As explored in an earlier draft of this paper, a similar logic applies if firms are funded through equity and equity investors are not fully diversified (Straub and Ulbricht, 2018).

be agnostic about the precise channel through which financial constraints impact a firm's activities and instead calibrate their impact directly based on existing evidence on firm behavior.

2.2 Equilibrium Characterization

We next characterize the equilibrium in the economy, starting with the price setting choice of intermediate-goods firms.

Price setting. Observe that, conditional on productivity $\mathcal{A}_{i,t}$, intermediate-goods firms solve a conventional monopolistic competition problem, charging a price with a constant markup over marginal cost,

$$p_{i,t} = \frac{\xi}{\xi - 1} \frac{w_t}{\mathcal{A}_{i,t}^{1/(\xi-1)}}.$$

With this price, operating profits are given by

$$\Pi_{i,t} = \xi^{-\xi} \left(\xi - 1\right)^{\xi-1} \mathcal{A}_{i,t} \frac{Y_t}{w_t^{\xi-1}}. \tag{10}$$

Technology choice. Given our assumptions, firms always prefer to operate the risky technology as long as banks are willing to fund it, and otherwise operate the baseline technology. To see when banks are willing to fund the risky technology, we rewrite the default condition (5) using (10),

$$\log A_{i,t} < \log\big(\left(1 + r_{i,t}\right)\phi\big) - \log\left(Y_t/w_t^{\xi-1}\right) + \log\left(\xi^{\xi}\left(\xi - 1\right)^{1-\xi}\right). \tag{11}$$

That is, firms with the risky technology default on their loan when $A_{i,t}$ falls below a certain threshold, which is more likely when the outstanding debt $(1 + r_{i,t})\phi$ is greater, the real wage w_t is greater, and aggregate demand Y_t is weaker. Using (11) together with the standard Normal cdf $\Phi(\cdot)$, we can express the probability of repayment, $1 - \mathbb{P}_t\big(\Pi_{i,t} < (1 + r_{i,t})\phi\big)$, in terms of the belief at the time of the funding choice, captured by $\mu_{i,t}$ and $\Sigma_{i,t}$. Substituting into (6), the zero-profit condition for risk loans becomes

$$\Phi\left(\frac{\mu_{i,t} - \log\big(\left(1 + r_{i,t}\right)\phi\big) + \log\left(Y_t/w_t^{\xi-1}\right) - \log\left(\xi^{\xi}\left(\xi - 1\right)^{1-\xi}\right)}{\sqrt{\Sigma_{i,t}}}\right) = \frac{1 + \lambda_t}{1 + r_{i,t}}. \tag{12}$$

If the zero profit condition holds for some $1 + r_{i,t} \geq 0$ and a firm has no bankruptcy flag, the firm receives funding for the risky technology; else it does not. Reformulating (12), we obtain the following result.

Proposition 1. *Define the* (risky) lending threshold *as*

$$\nu_t \equiv \log\left((1+\lambda_t)\,\phi\right) - \log\left(Y_t/w_t^{\xi-1}\right) + \log\left(\xi^{\xi}\,(\xi-1)^{1-\xi}\right).$$

Firm i obtains funding for the risky technology if and only if (i) it has no bankruptcy flag, and (ii) the belief $(\mu_{i,t}, \Sigma_{i,t})$ satisfies

$$\mu_{i,t} - \mathcal{V}\left(\Sigma_{i,t}\right) \geq \nu_t \tag{13}$$

where $\mathcal{V}(\Sigma)$ is defined as

$$\mathcal{V}\left(\Sigma\right) \equiv \min_{x\in(0,1]} \left\{\Phi^{-1}\left(x\right)\sqrt{\Sigma} - \log x\right\}.$$

Banks are willing to fund all risky projects for which $\mu_{i,t} - \mathcal{V}(\Sigma_{i,t})$ exceeds a time-varying threshold ν_t, which we henceforth refer to as *(risky) lending threshold*. We have $\mathcal{V}(0) = 0$ and $\mathcal{V}'(\Sigma) > 0$ for Σ that is not too large, capturing that default becomes more likely as uncertainty increases, which in turn increases default premia and reduces the willingness of banks to lend. Only in the pathological case where default is more likely than repayment, $\mathcal{V}$ may decrease in Σ. Henceforth, we assume that σ_ϵ is low enough so that $\mathcal{V}$ increases for $\Sigma \leq \sigma_\epsilon^2/(1-\rho^2)$, which is easily satisfied numerically for reasonable unconditional variances of log revenue productivity documented in the data.[11]

Belief dynamics. The cross-sectional distribution of beliefs $(\mu_{i,t}, \Sigma_{i,t})$ about productivities $A_{i,t}$ is a crucial state variable in our economy. From (3) we can derive the law of motion of beliefs about each firm i as

$$\mu_{i,t+1} = \begin{cases} \rho \log A_{i,t} + (1-\rho)\log \bar{A} & \text{if } i \in \mathcal{B}_t \\ \rho\mu_{i,t} + (1-\rho)\log \bar{A} & \text{if } i \notin \mathcal{B}_t \end{cases} \tag{14}$$

$$\Sigma_{i,t+1} = \begin{cases} \sigma_\epsilon^2 & \text{if } i \in \mathcal{B}_t \\ \rho^2\Sigma_{i,t} + \sigma_\epsilon^2 & \text{if } i \notin \mathcal{B}_t. \end{cases} \tag{15}$$

As long as a firm adopts the risky technology, learning about $A_{i,t}$ is perfect; uncertainty only reflects current innovations to $A_{i,t}$. By contrast, when a firm switches to the baseline technology, uncertainty about the productivity of its risky technology accumulates and beliefs converge towards the unconditional prior.

[11] More precisely, $\mathcal{V}$ is increasing on the relevant support, $[\sigma_\epsilon^2, \sigma_\epsilon^2/(1-\rho^2)]$, as long as $\sigma_\epsilon/\sqrt{1-\rho^2} < 0.7979$, which is easily satisfied given reasonable variances of log revenue productivities.

General equilibrium and steady state. Each firm i has an idiosyncratic state that is given by $\mathcal{S}_{i,t} \equiv (A_{i,t}, \mu_{i,t}, \Sigma_{i,t}, d_{i,t})$ where $d_{i,t} \in \{0,1\}$ is firm i's bankruptcy flag. In any given period, firm i's output and labor demand, $Y_{i,t}$ and $L_{i,t}$, are functions of its state $\mathcal{S}_{i,t}$ as well as of the aggregates (λ_t, w_t, Y_t),

$$Y_{i,t} = \mathcal{A}_{i,t}^{\xi/(\xi-1)} \left(\frac{\xi}{\xi-1}\right)^{-\xi} \frac{Y_t}{w_t^{\xi}} \qquad \text{and} \qquad L_{i,t} = \mathcal{A}_{i,t} \left(\frac{\xi}{\xi-1}\right)^{-\xi} \frac{Y_t}{w_t^{\xi}},$$

where $\mathcal{A}_{i,t}$ is firm i's technology, determined by Proposition 1. Aggregating across firms, we find that

$$w_t = (1-\xi^{-1})\boldsymbol{A}_t \qquad \text{and} \qquad Y_t = \boldsymbol{A}_t L_t \tag{16}$$

where

$$\boldsymbol{A}_t \equiv \left(\int_0^1 \mathcal{A}_{i,t}\,\mathrm{d}i\right)^{\frac{1}{\xi-1}} \tag{17}$$

corresponds to the efficiency wedge in the economy, in the spirit of Chari, Kehoe and McGrattan (2007), and $1-\xi^{-1}$ stems from the monopoly distortion induced by monopolistic competition. Together with the first order condition for household labor supply, $w_t = \upsilon L_t^{1/\zeta} C_t$, we find

$$(1-\xi^{-1})\boldsymbol{A}_t = \upsilon L_t^{1/\zeta}\left(\boldsymbol{A}_t L_t - \int_0^1 \phi_{i,t}\,\mathrm{d}i\right). \tag{18}$$

Conditional on firms' technology choices, this equation admits a unique positive solution for L_t. The solution always satisfies $\boldsymbol{A}_t L_t > \int_0^1 \phi\,\mathrm{d}i$. Thus, output Y_t is uniquely determined given $\boldsymbol{A}_t$.

However, in general equilibrium, the composition of the technology used by firms, and thus $\boldsymbol{A}_t$, is endogenous to Y_t and w_t. Specifically, $\boldsymbol{A}_t$ is a decreasing function of the lending threshold ν_t, which in turn is a function of Y_t and w_t (Proposition 1), capturing how in general equilibrium tighter credit constraints translate into suboptimal project choices. In Appendix B.3, we show that taking into account this feedback from Y_t and w_t to $\boldsymbol{A}_t$, (18) admits a unique fixed point as long as $\xi > 2$, which is satisfied in our calibration below. Conditional on this restriction and a given an initial distribution $\{\mathcal{S}_{i,0}\}$, the general equilibrium of our economy is unique, pinning down unique paths of aggregates, such as Y_t, L_t, w_t, as well as a unique path of the distribution of idiosyncratic states $\{\mathcal{S}_{i,t}\}$.

We next analyze the model from the perspective of a single firm, holding the wage w_t and aggregate output Y_t constant. Then, in Sections 4–7, we study how the general equilibrium of the model responds to an aggregate financial shock.

3 Endogenous Uncertainty and Lending

We are now ready to study the interaction between credit and learning that is at the core of our mechanism. In this section, we illustrate this interaction, focusing on the partial equilibrium dynamics of a single firm. From Proposition 1, a firm i is denied risky funding if

$$\mu_{i,t} - \mathcal{V}(\Sigma_{i,t}) < \nu_t, \tag{19}$$

where the lending threshold ν_t captures the combined impact of w_t, Y_t and the financial shock λ_t on the firm's access to credit. From (14) and (15), uncertainty about firms without risky funding at t increases at $t+1$; and expectations will remain anchored around $\mu_{i,t}$, slowly converging back to the unconditional mean, irrespective of the actual realization of $A_{i,t}$. Through this decoupling of belief dynamics from fundamentals, credit constraints get reinforced over time and may outlast any shock to ν_t.

3.1 Interaction Between Credit and Learning

To illustrate, suppose productivities are at their mean, $A_{i,s} = \bar{A}$; there is a constant lending threshold $\nu_s = \nu$ for all $s \geq 0$; firm i has no bankruptcy flag at $t = 0$; and the exogenous informative signal does not materialize. In that case, the joint dynamics of beliefs $(\mu_{i,t}, \Sigma_{i,t})$ are deterministic, and can be captured in a simple, yet useful, phase diagram. We show two examples of such phase diagrams in Figure 1, for two different values of ν.[12]

The thin gray line depicts the contour along which (19) holds with equality, dividing the state space into a region where the firm has access to risky bank loans (southeastern) and one where it does not (northwestern). The red line corresponds to the constant expectations locus, where $\mu_{i,t} = \mu_{i,t-1} = \log \bar{A}$. The blue lines correspond to the constant uncertainty locus, where $\Sigma_{i,t} = \Sigma_{i,t-1}$.

The uncertainty locus consists of two separate pieces. First, when uncertainty is at its lowest, $\Sigma_{i,t} = \sigma_\epsilon^2$, and the expected risky productivity $\mu_{i,t}$ is sufficiently large for funding, $\mu_{i,t} > \mathcal{V}(\sigma_\epsilon^2) + \nu$, uncertainty remains constant at σ_ϵ^2. This is the flat blue line at the bottom of Figure 1. Second, and symmetrically, when uncertainty is at its highest, $\Sigma_{i,t} = \sigma_\epsilon^2/(1-\rho^2)$, and $\mu_{i,t}$ is sufficiently low, $\mu_{i,t} < \mathcal{V}\left(\sigma_\epsilon^2/(1-\rho^2)\right) + \nu$, there is no funding and uncertainty remains at its highest level. For intermediate levels of uncertainty, the firm is marginally funded when $\mu_{i,t} - \mathcal{V}(\Sigma_{i,t}) = \nu$. Starting from this curve, slightly greater uncertainty or slightly more pessimistic expectations raise uncertainty, while points on or below the curve

[12] In the left panel, ν is set to the value it takes at the aggregate steady state, ν^{ss}, given the parameterization in Section 4.1. In the right panel, it is set to $\nu^{\text{ss}} + 0.1$.

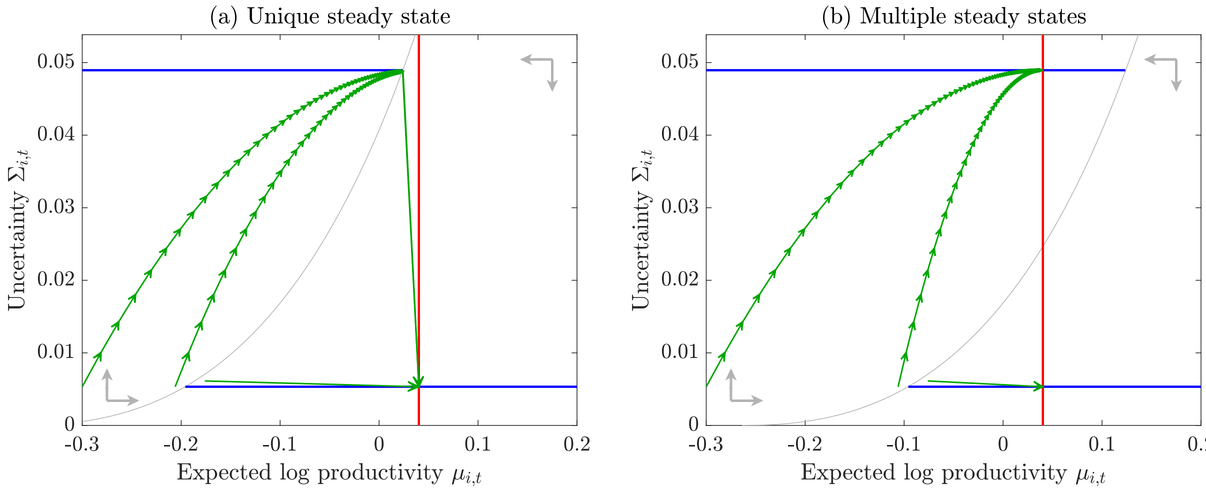

Figure 1: Phase diagram for firm-level beliefs in the absence of shocks

Note. Thin gray lines depict ($\mu - V(\Sigma) = \nu$)-contours; Z-shaped blue lines are the constant-$\Sigma_{i,t}$ locus; vertical red lines are the constant-$\mu_{i,t}$ locus. Arrowheads represent one period in time along the plotted trajectories. Parameterization as in Section 4.1; c.f. Footnote 12. *Left:* Case with a unique steady state ($\nu < \overline{\nu}$). *Right:* Case with multiple steady states ($\underline{\nu} < \nu < \overline{\nu}$).

reduce uncertainty.

The "Z" shaped pattern visible in Figure 1 captures the self-reinforcing nature of endogenous uncertainty in our model. When uncertainty is high today, a firm is less likely to receive funding for the risky technology, which further increases uncertainty going forward. When ν is neither too low nor too high, this effect can be sufficiently strong to generate two steady states in the phase diagram. As our next proposition shows, and Figure 1 illustrates, this happens when the Z-shape intersects with the vertical red line.

Proposition 2. *There exist two thresholds $\underline{\nu} < \overline{\nu}$, such that for all $\underline{\nu} \leq \nu \leq \overline{\nu}$, there are two (non-stochastic) steady states at the firm-level, and otherwise there is a unique (non-stochastic) steady state. The thresholds are given by $\underline{\nu} = \log \bar{A} - \mathcal{V}(\sigma_\epsilon^2)$ and $\overline{\nu} = \log \bar{A} - \mathcal{V}(\sigma_\epsilon^2/(1-\rho^2))$.*

For intermediate levels of the lending threshold ν, a denial in funding for the risky technology is infinitely persistent in the absence of shocks. Accordingly, a one-time disruption in a firm's access to risky funding can cut it off indefinitely from future risky funding. This is certainly a stylized result, but it neatly illustrates the forces that are active in the model. Of course, even when the steady state is unique, disruptions in credit access may be persistent (though not indefinite), as illustrated by the gray trajectories in the left panel of Figure 1. Along these trajectories, each arrowhead represents one period. Accordingly, the distance between two consecutive arrowheads is an inverse measure of the speed at which beliefs are evolving.

The three trajectories differ in the persistence of beliefs and the amount of uncertainty induced along the path. Along the rightmost trajectory, the firm is initially unconstrained and beliefs immediately adjust to the unique steady state. By contrast, along the two trajectories starting to the left of the gray contour line, the firm is initially denied risky funding so that learning breaks down. Accordingly, mean beliefs $\mu_{i,t}$ only slowly converge to the unconditional prior, whereas uncertainty accumulates to higher and higher levels as information about past levels of $A_{i,t}$ becomes less and less useful for predicting current productivity. This, in turn, reinforces tight credit constraints. Hence, even though the steady state is unique, a firm can find itself lacking full access to credit for a significant period of time, unable to invest in their risky technology.

More generally, the duration without access to risky funding is governed by a "race" between the mean-reversion in $\mu_{i,t}$ and rising uncertainty. Consider a marginally constrained firm with $\mu_{i,t}$ just below $\mathcal{V}(\sigma_\epsilon^2) + \nu$. Stepping forward in time by one period, it will be constrained at $t+1$ if and only if

$$\rho\mathcal{V}(\sigma_\epsilon^2) - \mathcal{V}((1+\rho^2)\sigma_\epsilon^2) < (1-\rho)(\nu - \log \bar{A}).$$

Hence, the marginally constrained firm will lose access to credit for multiple periods if either aggregate credit conditions are sufficiently bad (ν is sufficiently large) or if $A_{i,t}$ is sufficiently persistent (ρ is sufficiently large).

3.2 Temporary Disruption in Credit

The phase diagram in Figure 1 studied permanent shocks to ν_t. We next relax this assumption and study the average effect of a one-time shock to ν_t, first in a phase diagram, then as a simulation in our model that includes idiosyncratic shocks such as the exogenous revelation of information with probability θ.

To do so, we fix an initial productivity A and lending threshold ν, such that the firm is just active absent a shock, $\rho \log A = \nu + \mathcal{V}\left(\sigma_\epsilon^2\right)$. Now suppose that at $t = 0$, i's access to risky funding is curtailed by an exogenous increase in the lending threshold to $\nu_0 > \nu$, which mechanically reverts back to ν at $t = 1$. For concreteness, we interpret the shift in ν_0 as stemming from a financial shock λ_0, but we note that in general equilibrium ν_t also reflects shifts in aggregate demand Y_t and wages w_t.

Figure 2 illustrates the dynamics using the phase diagram developed above. In the diagram, the shock to the banking sector results in a rightward-shift of the $(\mu - \mathcal{V}(\Sigma) = \nu)$-contour (the dashed gray line). For sufficiently large ν_0, the firm is denied funding for the risky technology, setting in motion a feedback loop between uncertainty and continued inability to fund the risky technology. Once uncertainty has passed the original $(\mu - \mathcal{V}(\Sigma) = \nu)$-contour line (the

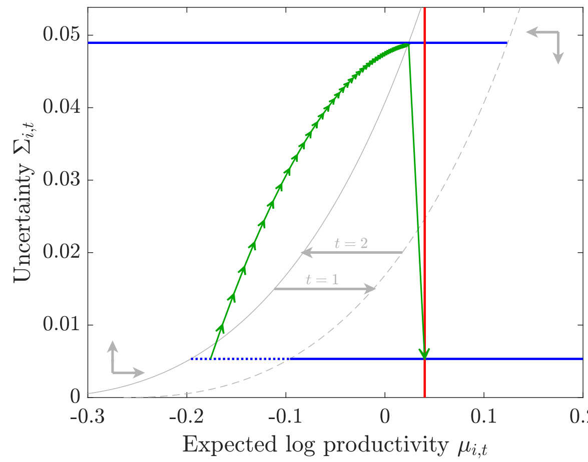

Figure 2: Dynamic response to a financial shock at $t = 1$ and a subsequent recovery at $t = 2$

Note. Arrowheads represent one period in time along the plotted trajectory. Parameterization as in Section 4.1, with $\nu_0 = \nu_{2+s}$, $s \geq 0$, set to the value of ν at the aggregate steady state, and $\nu_1 = \nu^{\text{ss}} + 0.1$.

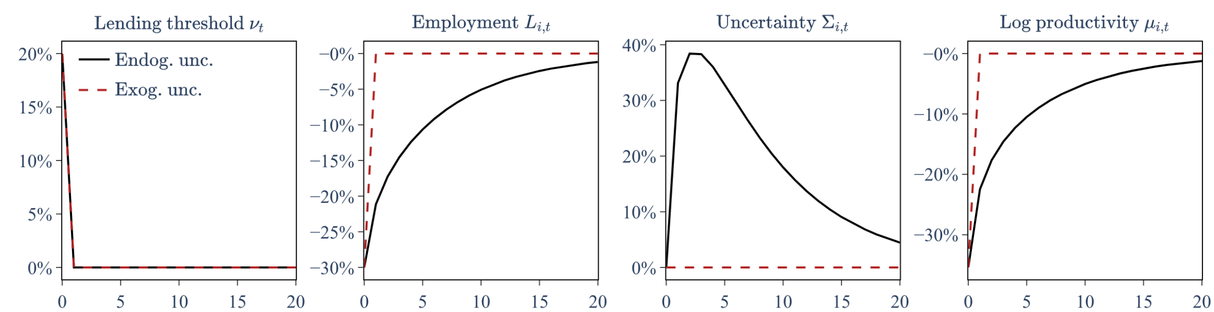

Figure 3: Impact of temporary financial shock on firm dynamics

Note. Black solid line: Effect of one time disruption in credit, $\nu_0 > \nu$, in period $t = 0$ on the average evolution of a firm close to the funding threshold, $\rho \log A = \nu + \mathcal{V}\left(\sigma_\epsilon^2\right)$. Red dashed line: Same evolution, but fixing uncertainty exogenously at $\Sigma_{i,t} = \sigma_\epsilon^2$. Parameterization as in Section 4.1.

solid gray line), even a reversal of ν_t to ν does not end the feedback loop, generating internal persistence of the shock.[13]

Figure 3 repeats the experiment in our model with all firm-level shocks active, showing how the average evolution across different sample paths is affected by a one-period long disruption in credit. To isolate the contribution of the endogenous-uncertainty channel, we contrast the model's response (solid black lines) with a counterfactual response, in which the firm suffers the same exogenous financial shock but uncertainty is fixed at its lower bound, $\Sigma = \sigma_\epsilon^2$ (dashed red lines). We call this the *exogenous uncertainty* model as a contrast with

[13]Here we initialized the firm close enough to the constraint so that uncertainty surpasses the original $(\mu - \mathcal{V}(\Sigma) = \nu)$-contour line after one period. In general, an exogenous disruption in credit lasting for $T-1$ periods cause internal persistence beyond the exogenous shock if $\rho^T \log A < \nu_0 + \mathcal{V}\left(\frac{1-\rho^{2T}}{1-\rho^2}\sigma_\epsilon^2\right)$.

our *endogenous uncertainty* model. The exogenous uncertainty model will serve as a useful benchmark for the remainder of this paper.

In both cases, output initially drops due to the switch in technologies for the duration of the financial shock. The difference between our model and the exogenous-uncertainty counterfactual emerges at $t = 1$. Whereas output recovers in the counterfactual once access to credit is restored, the firm continues to be denied funding in the presence of endogenously increased uncertainty. The disruption in credit continues until either $\mu_{i,t}$ crosses the $(\mu - \mathcal{V}(\Sigma) = \nu)$-contour in Figure 2 or the potential productivity $A_{i,t}$ is exogenously revealed (with probability θ). In both cases, uncertainty drops to σ_ϵ^2 and the firm switches back to the risky technology.

The dynamics shown in Figure 3 are reminiscent of the evidence in Huber (2018), who shows that a quasi-exogenous temporary financial shock can have a long-lasting effect on firm performance. In particular, Huber (2018) shows that the gap in employment between firms that were exposed to the shock and firms that were not remains elevated for two years after the shock.

3.3 Informational Externalities

We conclude this section with a brief discussion of efficiency. Our specification of credit constraints implies two sources of inefficiency. First, credit access is *statically inefficient* due to the presence of default costs, which give rise to the usual static wedge between supply and demand for credit.[14] Second, the combination of endogenous learning and external funding introduces a novel *dynamic inefficiency* that arises because atomistic banks do not internalize the option value of learning about a firm's risky technology. In our setup, this is because firms and banks cannot write contracts that are contingent on productivity realizations in future periods. This leads banks to lend too little.

The two inefficiencies suggest welfare gains from subsidizing bank lending. Interestingly, by mitigating the dynamic inefficiency, subsidized bank lending generates new information about firms' risky productivities $A_{i,t}$, helping the market identify which firms are creditworthy. Hence, public lending may in fact crowd *in* future private lending, raising the social returns.

To illustrate the two inefficiencies, we numerically solve for the steady state distribution of firms given the parameterization from Section 4.1, and compute the private and social gains from lending to firms with various initial expectations $\mu_{i,t}$ at the lowest uncertainty level $\Sigma_{i,t} = \sigma_\epsilon^2$ at some time t. Specifically, for each $\mu_{i,t}$, we compute: (i) the private benefit

[14] In general, as long as banks cannot recover the full share of operating profits in the event of default, the maximal repayment that banks can generate (under any interest rate) is strictly smaller than $\mathbb{E}_t[\Pi_{i,t}]$ and, hence, the statically marginally profitable, risky project with $\mathbb{E}_t[\Pi_{i,t}] \to \phi$ will never be funded by banks.

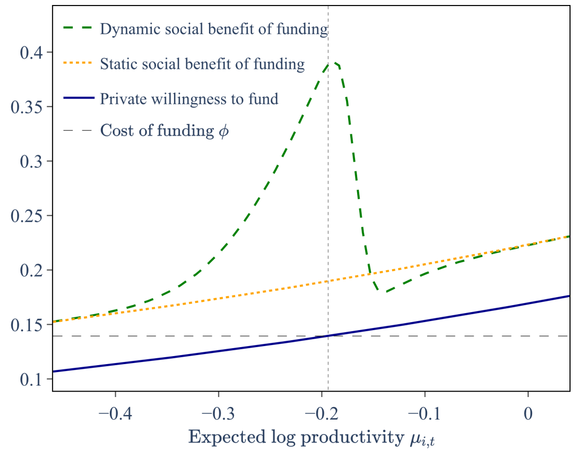

Figure 4: Private and social benefits from lending

Note. The vertical dotted line indicates the equilibrium threshold $\nu + \mathcal{V}(\sigma_\epsilon^2)$ for lending. Parameterization as in Section 4.1.

from lending as the largest risky loan a bank would be willing to lend, given by

$$\phi_{i,t}^{\text{private}} = \phi \exp\left\{\mu_{i,t} - \mathcal{V}\left(\sigma_\epsilon^2\right) - \nu\right\},$$

(ii) the static social benefit from risky lending, given by

$$\Delta\Pi_{i,t}^{\text{static}} = \tilde{\phi} + \left(\mathbb{E}\left[A_{i,t}|\mu_{i,t}, i \in \mathcal{B}_t\right] - \tilde{A}\right) \xi^{-\xi}\left(\xi - 1\right)^{\xi-1} \frac{Y}{w^{\xi-1}},$$

and (iii) the dynamic social benefit from risky lending,

$$\begin{aligned}\Delta\Pi_{i,t}^{\text{dynamic}} &= \Delta\Pi_{i,t}^{\text{static}} + \\ &+ \sum_{s=1}^{\infty}(1+r)^{-s}\Big(\mathbb{E}\left[\mathcal{A}_{i,t+s}|\mu_{i,t}, i \in \mathcal{B}_t\right] - \mathbb{E}\left[\mathcal{A}_{i,t+s}|\mu_{i,t}, i \notin \mathcal{B}_t\right]\Big) \xi^{-\xi}\left(\xi - 1\right)^{\xi-1} \frac{Y}{w^{\xi-1}} \\ &\qquad - \sum_{s=1}^{\infty}(1+r)^{-s}\Big(\mathbb{E}\left[\phi_{i,t+s}|\mu_{i,t}, i \in \mathcal{B}_t\right] - \mathbb{E}\left[\phi_{i,t+s}|\mu_{i,t}, i \notin \mathcal{B}_t\right]\Big).\end{aligned}$$

After time t, firms follow their equilibrium behavior. We note that the static social benefit from lending corresponds to the increase in date-t utility (in units of date-t consumption) from lending to firm i at date t. The dynamic social benefit from lending corresponds to the increase in welfare across all future periods (in units of date-t consumption) from lending to firm i at date t.

Figure 4 plots $\phi_{i,t}^{\text{private}}$, $\Delta\Pi_{i,t}^{\text{static}}$, and $\Delta\Pi_{i,t}^{\text{dynamic}}$ against $\mu_{i,t}$. As expected, the maximum private loan size exceeds the fixed cost ϕ precisely to the right of the lending threshold (vertical dotted line), where $\mu_{i,t} > \nu + \mathcal{V}(\sigma_\epsilon^2)$. The static social benefit from lending $\Delta\Pi_{i,t}^{\text{static}}$ lies consistently above the private willingness to fund, illustrating the standard static inefficiency induced by default risk. The dynamic social benefit $\Delta\Pi_{i,t}^{\text{dynamic}}$ is closely aligned with the static benefit for productivity expectations $\mu_{i,t}$ far away from the threshold. For those, the actual technology used in later periods $\mathcal{A}_{i,t+s}$ is close to independent of whether in period t the firm was funded, $i \in \mathcal{B}$, or not, $i \notin \mathcal{B}$.

For values of $\mu_{i,t}$ right around the lending threshold, however, there is a large dynamic gain. Information revelation in period t on a firm's risky productivity $A_{i,t}$ allows banks to accurately assess on which side of the funding threshold a firm's future expectation $\mu_{i,t+1}$ lies. This leads to more accurate lending by banks in the future and therefore increases the present discounted value of public lending.[15]

4 Aggregate Credit Crunches

Having studied the impact of a financial shock on a single firm, we next study the general equilibrium response of the economy to an aggregate financial shock.

4.1 Parameterization

We interpret one period as a quarter, and set the discount factor β to 0.99. The Frisch elasticity of labor supply ζ is set to 2, the elasticity of substitution between consumption goods is set to 5, and the scaling parameters υ and $\bar{A}$ are set to normalize steady state employment and output to 1. The revenue productivity parameters are set to $\rho = 0.9440$ and $\sigma_\epsilon = 0.0726$, consistent with the revenue productivity process estimated by Foster, Haltiwanger and Syverson (2008) (converted to a quarterly frequency). Our choice for the baseline productivity $\tilde{A}$ is based on Chodorow-Reich (2013, online appendix). Using bank lending relations to instrument for the credit access of firms, Chodorow-Reich finds that an exogenous preclusion from credit results in an employment decline of 53%, but qualifies that this number is likely overstating the true effect due to the first-stage coefficients being biased towards zero. Keeping in mind this qualification, we set $\tilde{A}$ so that the marginal employment effect of having access to risky credit equals 30%.

It remains to choose values for ϕ, η and θ. We set ϕ to pin down the share of firms without access to risky funding at the steady state. Lacking more precise data, we inform

[15]The region in which the dynamic gains lie below the static one indicates firms for which lending today may reveal that their productivity lies below the lending threshold in the future, whereas absent public lending access to credit markets would be restored soon, reducing the dynamic gains from lending.

Table 1: Calibrated parameters

Parameter	β	ζ	ξ	υ	$\bar{A}$	$\tilde{A}/\bar{A}$	ρ	σ_ϵ	ϕ	η	θ
Endog. uncertainty	0.99	2.000	5.000	0.913	1.041	0.563	0.944	0.073	0.139	0.350	0.117
Exog. uncertainty	0.99	2.000	5.000	0.907	0.983	0.563	0.944	0.073	0.121	0.350	1.000

our choice of ϕ using two proxies for the fraction of firms that lack sufficient funding. First, among Compustat firms in the years 1976–1999, the sales-weighted average fraction of firms that does not pay dividends is 8%.[16] Second, among small businesses, the 2020 Fed Small Business Credit Survey documents that 21% of all firms were (partially) denied credit and an additional 9% of firms were discouraged from seeking credit, because they believed they would be turned down. Based on this evidence, we set ϕ so that 25% of firms do not have access to risky funding at the steady state, implying a sales-weighed average of 8%. Appendix D.2 explores alternatives. Given ϕ, we then set $\tilde{\phi}$ in proportion to the expected productivity ratio, $\tilde{\phi} = (\tilde{A}/\bar{A})\phi$. Next, we set the rate of bankruptcy removals η to 0.35, targeting an average bankruptcy length of 2.86 quarters, consistent with the average duration of Chapter 11 bankruptcy negotiations documented by Teloni (2015).[17] Finally, we set the exogenous information revelation rate θ to implement an average duration of a denial in risky funding of 2 years, consistent with the impact on firm growth documented in Huber (2018).

Below, we compare the endogenous uncertainty model with an exogenous uncertainty counterfactual where $\theta = 1$; that is, uncertainty $\Sigma_{i,t}$ always remains at σ_ϵ^2. Moving to exogenous uncertainty affects the steady state of the model as well as the model's shock propagation. Since our focus is on the role of endogenous uncertainty in generating endogenous persistence, we make the exogenous uncertainty model as comparable as possible by calibrating ϕ in order to hit the same on-impact response of output to a financial shock. The other parameters are chosen in the same way as above; e.g., the average level of productivity $\bar{A}$ is set to normalize steady state output to 1. Appendix D.3 considers two alternative ways to calibrate the exogenous uncertainty counterfactual.

Table 1 summarizes the calibrated parameters. We solve the model in the sequence space (Boppart, Krusell and Mitman, 2018; Auclert et al., 2021); for details, see Appendix C.

[16] We only use data until 1999, excluding the burst of the dot-com bubble and the 2008/09 financial crisis, in line with our interpretation of the steady state as normal times.

[17] The targeted duration is documented for 2005–2014, after the Bankruptcy Abuse Prevention and Consumer Protection Act went into effect, and includes pre-negotiated deals.

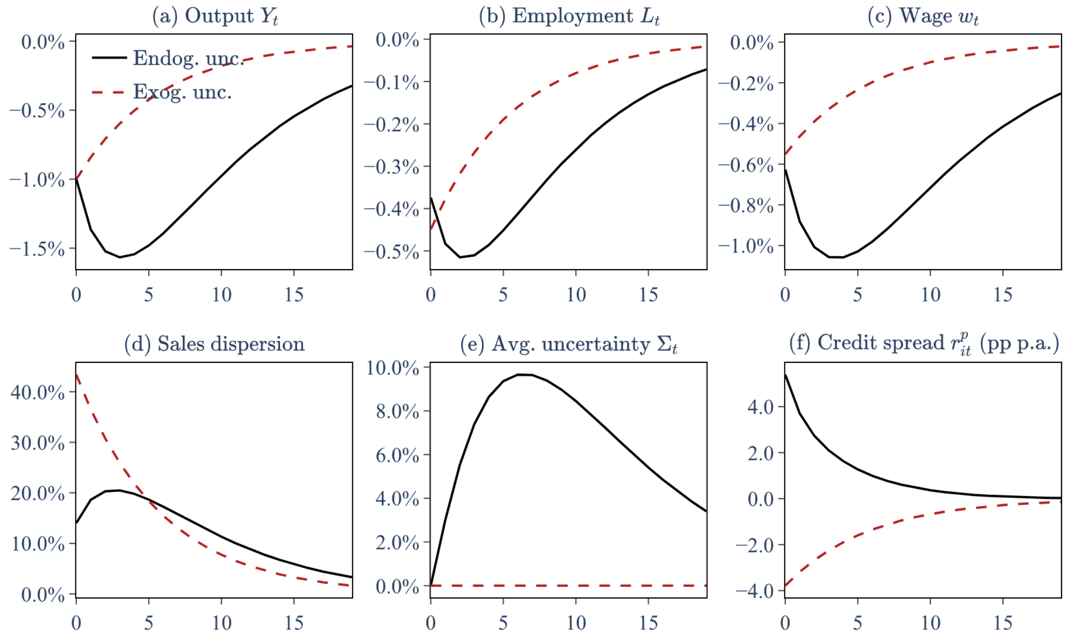

Figure 5: General equilibrium response to an AR(1) financial shock

Note. Panels (a)–(e) are denominated in percentage deviations from the steady state; panel (f) shows the increase in credit spreads of the 10% least productive firms that still received funding relative to the steady state denominated in percentage points per annum. Solid black lines are model responses; dashed red lines are exogenous uncertainty counterfactuals where uncertainty is fixed at $\Sigma_{i,t} = \sigma^2_\epsilon$ for all i and t.

4.2 Simulation of an Aggregate Financial Shock

We are now ready to explore the economy's response to an aggregate financial shock. The economy is initialized at its stochastic steady state where $\lambda_{t-s} = 0$ for $s > 0$ and the cross-section of firms is at its ergodic distribution. We simulate the economy's response to a financial shock λ_t to first order in aggregate variables (remaining fully nonlinear in idiosyncratic variables), as, e.g., in Reiter (2009), Boppart, Krusell and Mitman (2018) and Auclert et al. (2021). We assume λ_t follows an AR(1) process with a half-life of four quarters (implying an autocorrelation of 0.84). The magnitude of the financial shock is normalized to induce a 1% drop in output at impact.

Figure 5 displays the model's responses to the perturbation at $t = 0$, depicted by solid black lines. The dashed red line shows the exogenous uncertainty counterfactual, where uncertainty is fixed at $\Sigma_{i,t} = \sigma^2_\epsilon$ for all i and t. The responses in panels (a)–(e) are in percentage deviations from their respective steady state values; the response in panel (f) shows the increase in the credit spread of the 10% weakest funded firms relative to steady state in percentage points.

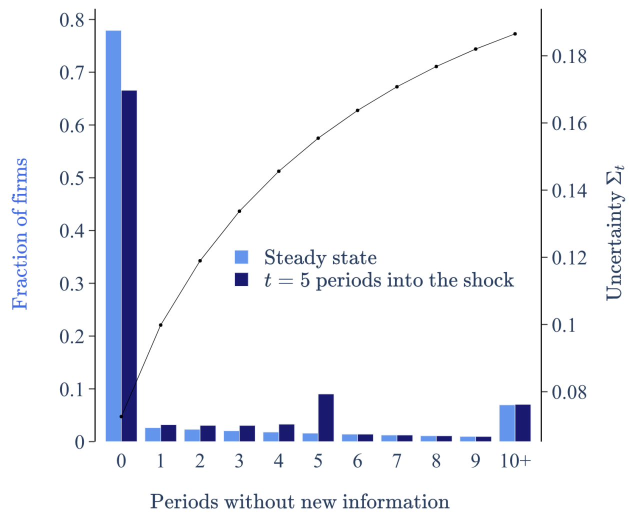

Figure 6: The distribution of uncertainty 5 periods into the impulse response and at the steady state

Note. Solid black line: level of uncertainty corresponding to s periods without new information (right axis).

Amplification and persistence. Because uncertainty is predetermined, the impact response at date 0 is fully explained by tightened credit constraints. Starting at date 1, however, the adverse effects of rising uncertainty start to both amplify and prolong the crisis relative to the exogenous uncertainty case. At $t = 4$, output in the exogenous uncertainty counterfactual has recovered exactly half of its impact losses. By comparison, at $t = 4$, output in the endogenous uncertainty economy is more than 1.5 percent below steady, surpassing the impact effect. In terms of half-lives, recovery takes four times as long in the endogenous uncertainty economy (16 quarters) compared to the exogenous uncertainty counterfactual (4 quarters).

The main reason for the persistent increase in uncertainty is the feedback between uncertainty and lending we laid out in Section 3. As the financial shock hits the economy, some firms loose access to funding for the risky technology. This can lead to several periods without new information, and hence greater uncertainty. Figure 6 illustrates this at $t = 5$, plotting the cross-sectional distribution of uncertainty. Compared to the steady state distribution, there are increased masses of firms in the states corresponding to 1–5 periods without new information. Those states are associated with significantly greater uncertainty (right y-axis), raising the overall level of uncertainty. Particularly striking is the "spike" of mass in firms who lost funding at the start of the crisis and have not regained credit as of $t = 5$. This reflects the endogenous persistence of financial shocks arising at the micro-level (see Section 3), which carries over and generates a significant amount of persistence in the impulse responses in Figure 5.

Credit spreads, default rates, and dispersion. Rising uncertainty also helps explain a few financial market characteristics typically associated with financial crises. First, rising uncertainty increases average credit spreads, defined as before as the gap between the rate charged to firms and the internal discount rate by banks (which is equal to the lending rate on safe loans), $1 + r^p_{i,t} = (1 + r_{i,t})/(1 + \lambda_t)$. On average, as most firms remain far away from the lending threshold, the credit spread only increases by 40 basis points or so. For firms that lose funding, the implied increase is prohibitively large. However, even the weak *funded* firms experience a strong increase in credit spreads. Panel (f) of Figure 5 shows that the average credit spread of the 10% least productive firms that still receive funding increases by over four percentage points.

Second, default rates increase as uncertainty rises over the course of the response. Third, consistent with the evidence in Bloom et al. (2018), increased uncertainty at the firm-level translates into an increased cross-sectional dispersion of firm sales, plotted in panel (d) of Figure 5. This is caused by an increase in firms that are unable to finance the risky technology, and therefore have to resort to using the baseline technology. The increase is present both with and without endogenous uncertainty but is somewhat more persistent in the endogenous uncertainty model.[18]

Disagreement. With a few extra ingredients, our model also has testable predictions for the beliefs of market observers such as professional forecasters. Intuitively, as the publicly available information about firms that are denied risky lending diminishes, market observers will rely more on other sources to form their beliefs. As long as those other sources are partially dispersed across observers, disagreement among market observers increases when an increasing number of firms becomes constrained.

To formalize this prediction, consider a set of outside observers (or forecasters) $j \in [0, 1]$. In addition to $\mathcal{I}_t$, these forecasters each observe a private signal $\omega_{ij,t} = \log A_{i,t} + \psi_{ij,t}$, where $\psi_{ij,t}$ is normally distributed with zero mean and variance σ^2_ψ, i.i.d. across i, j, and t. For simplicity, we assume that forecaster do not interact with the rest of the economy and that in each period the previous generation of forecasters is replaced by a new one.[19] The belief of forecaster j about firm i's productivity at date t is given by

$$\tilde{\mu}_{ij,t} = \frac{\Sigma^{-1}_{i,t}\mu_{i,t} + \sigma^{-2}_\psi \omega_{ij,t}}{\Sigma^{-1}_{i,t} + \sigma^{-2}_\psi}. \tag{20}$$

[18] Gourio (2014) also offers an explanation for countercyclical default and a countercyclical dispersion of firm sales.

[19] This assumption spares us from dealing with infinite regress (Townsend, 1983). Nevertheless it is worth pointing out that our predictions about disagreement are not limited to short-lived forecasters, but would more generally carry over to any set of agents with dispersed signals (or priors).

In Appendix A, we use these forecasters' beliefs to compare the model's predictions with micro data from a survey of professional forecasters. From (20), the degree of "disagreement" among forecasters is given by

$$\mathrm{sd}_j[\tilde{\mu}_{ij,t}] = \frac{\sigma_\psi^{-1}}{\Sigma_{i,t}^{-1} + \sigma_\psi^{-2}}. \tag{21}$$

Thus, according to our model, there should be a tight empirical link between disagreement and the degree to which a firm is financially constrained. As shown in Appendix A, this is indeed the case.

5 Extensions

We next present three extensions that demonstrate how our mechanism operates (i) in the presence of investment and capital, (ii) in a New Keynesian version of our model, and (iii) when some firms do not rely on external funds to finance their projects.

5.1 Introducing Capital

Our first extension introduces capital to the baseline model in Section 2 and compares it to a standard real business cycle (RBC) model. To do so, we modify the production function of firm i to a Cobb-Douglas aggregate of capital and labor

$$Y_{i,t} = \mathcal{A}_{i,t}^{\frac{1}{\xi-1}} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha},$$

where capital $K_{i,t}$ is rented at the competitive rate $1 + r_t^K > 0$ from households. The representative household is now allowed to not only save in bonds B_t (which are still in zero net supply) but also in capital K_t. The date-t budget constraint now reads

$$C_t + B_{t+1} + K_{t+1} = w_t L_t + (1 + r_{t-1}) B_t + \left(1 + r_t^K - \delta\right) K_t + T_t.$$

As usual, capital K_t is determined one period in advance. Market clearing,

$$K_t = \int_0^1 K_{i,t} \, \mathrm{d}i,$$

determines the rental rate $1 + r_t^K$ in equilibrium. All other model elements remain unchanged. Households still maximize utility (7), firms maximize profits (4) and require a bank loan to finance the operating cost $\phi_{i,t}$. The financing condition (13) still applies, only that the lending threshold ν_t is now given by

$$\nu_t = \log\left((1+\lambda_t)\,\phi\right) - \log\left(\frac{Y_t}{(1+r_t^K)^{\alpha(\xi-1)} w_t^{(1-\alpha)(\xi-1)}}\right) + \log\left(\xi^{\xi}\,(\xi-1)^{1-\xi}\right).$$

We next show that the model with capital is equivalent to an RBC model, with an endogenous process for TFP corresponding to the efficiency wedge introduced in (17) and an endogenous process for a resource wedge as defined below.

Proposition 3. *Conditional on processes of the efficiency wedge $\{\boldsymbol{A}_t\}$, defined in (17), and a resource wedge $\{G_t\}$, defined by*

$$G_t \equiv \int_0^1 \phi_{i,t}\,\mathrm{d}i, \tag{22}$$

the equilibrium behavior of $\{C_t, K_t, L_t\}$ (and therefore also of other aggregates, such as Y_t, w_t, r_t^K) is described by a standard RBC model,

$$C_t^{-1} = \mathbb{E}_t\left[\beta\left((1-\xi^{-1})\alpha\boldsymbol{A}_{t+1}K_{t+1}^{\alpha-1}L_{t+1}^{1-\alpha} + 1 - \delta\right)C_{t+1}^{-1}\right] \tag{23}$$

$$\upsilon L_t^{1/\zeta} = (1-\xi^{-1})\,(1-\alpha)\,C_t^{-1}\boldsymbol{A}_t K_t^{\alpha} L_t^{-\alpha} \tag{24}$$

$$C_t + G_t + K_{t+1} = \boldsymbol{A}_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)\,K_t, \tag{25}$$

with monopoly distortion $1-\xi^{-1}$.

Proposition 3 is intuitive. The path of the lending threshold $\{\nu_t\}$ shapes the process of the distribution of productivities $\{\mathcal{A}_{i,t}\}$ and fixed costs $\{\phi_{i,t}\}$. Given $\{\mathcal{A}_{i,t}, \phi_{i,t}\}$, the process of the efficiency and resource wedges $\{\boldsymbol{A}_t, G_t\}$ can be computed from (17) and (22). The lending threshold ν_t itself is determined by the interacting financial and information frictions. The financial shock λ_t acts by shifting ν_t.

However, as before, ν_t is also a function of output Y_t. This is especially relevant here, as it generates a two-way feedback between ν_t and the capital stock K_t. A tighter lending threshold ν_t reduces the efficiency wedge $\boldsymbol{A}_t$ going forward, as more firms have trouble obtaining bank funding. Reduced efficiency, in turn, leads to reduced investment in capital, which further tightens the lending threshold (raising ν_t).

Figure 7 illustrates these dynamics for the same financial shock as in Figure 5 and compares the responses to the ones in the baseline model without capital.[20] While the responses in the economy with exogenous uncertainty are still modest, the endogenous uncertainty economy

[20]We choose $\alpha = 0.30$ and a quarterly depreciation rate of $\delta = 0.02$.

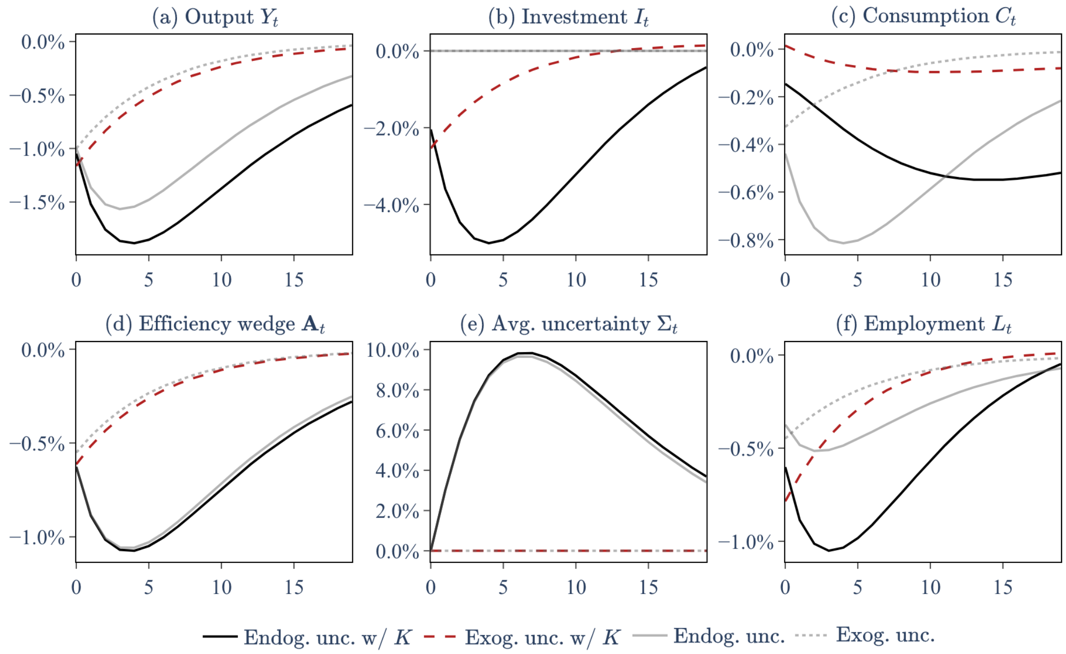

Figure 7: General equilibrium response in the model with capital

generates more amplification and persistence than before. The reason why is the strong reduction in investment in Panel (b).

5.2 Endogenous Uncertainty and Aggregate Demand

Next, we explore an extension of our model where we introduce nominal rigidity and households are exposed—in a stylized form—to uninsurable income risk. With these modifications, aggregate demand will sharply drop in response to a financial shock, which in turn tightens lending standards through its impact on ν_t as defined in Proposition 1.

Specifically, to expose aggregate demand to non-trivial fluctuations, we assume that half of all households are hand-to-mouth with $C_t^{\text{htm}} = w_t L_t/2$, whereas the remainder of households behave as in our baseline model, maximizing utility (7) subject to the budget constraint (8), only with labor income given by $w_t L_t/2$. In order for those demand fluctuations to have a real effect, we further assume nominal wage rigidities and a monetary authority that does not implement the flexible-price allocation. Wage rigidities are modeled as in Hagedorn, Manovskii and Mitman (2019) and Auclert, Rognlie and Straub (2018), with an equal-rationing assumption on hours across the two types of agents. Following Woodford (2011), the exact formulation of wage rigidity does not matter for real variables when the monetary authority stabilizes the real interest rate at its steady state level, $r_t = r_{\text{ss}}$, which is

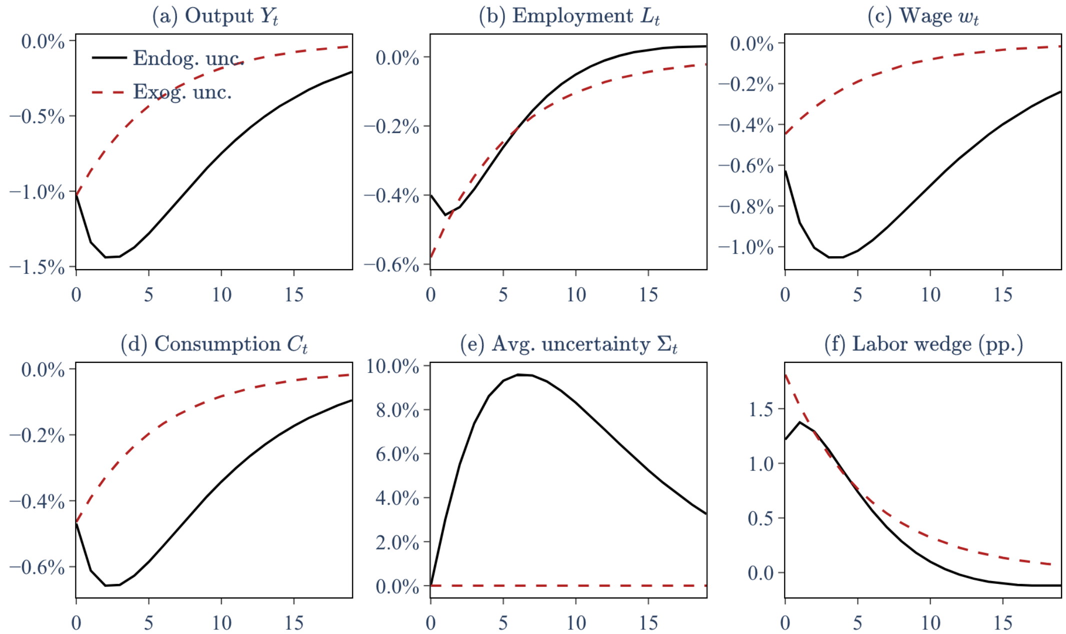

Figure 8: Response to financial shock with nominal rigidities and hand-to-mouth agents

Note. All parameters as in Section 4.1. Share of hand-to-mouth agents of 50%.

an assumption we make here as well.[21] Our results here would qualitatively be very similar with an active Taylor rule, though quantitatively would depend on the flexibility of nominal wages.

Aggregate consumption in this model is then characterized by

$$C_t = C_t^{\text{base}} + w_t L_t / 2$$
$$(C_t^{\text{base}})^{-1} = \mathbb{E}_t \left[\beta \left(1 + r_{\text{ss}}\right) (C_{t+1}^{\text{base}})^{-1} \right].$$

With $C_t^{\text{base}} \to C_{ss}^{\text{base}}$ for $t \to \infty$, and $\beta(1 + r_{\text{ss}}) = 1$, this automatically implies that C_t^{base} is indeed constant at its steady state level. Fluctuations in aggregate consumption therefore come from changes in labor income. Conditional on $\{\mathcal{A}_{i,t}, \phi_{i,t}\}$, which are determined by the same equations as in our baseline model, the competitive equilibrium in this model is then determined by C_t along with (9), (16) and (17).

Figure 8 simulates the resulting dynamics in response to the same financial shock as in Figure 5, and compares them again to the exogenous uncertainty model. The impulse responses look very similar even though the financial shock propagates differently in this

[21] A stable real interest rate follows, for example, from a Taylor rule with a unit coefficient on expected inflation. As in Woodford (2011) the natural equilibrium selection is the one assuming output returning to its natural steady state level as $t \to \infty$.

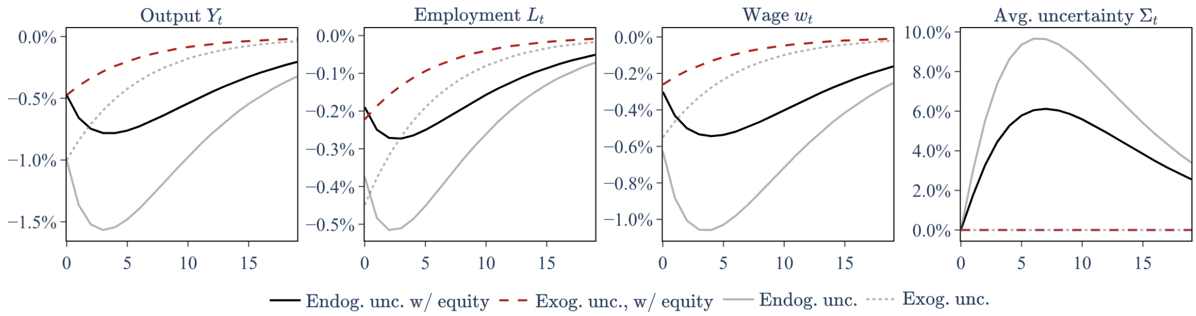

Figure 9: General equilibrium response in the model with both bank-financed and equity-financed firms

economy with nominal rigidities. By throttling new loans to firms, the financial shock directly reduces spending of firms and thus aggregate demand and aggregate income. This is then amplified via the Keynesian cross as hand-to-mouth households cut back on their spending in response to lower incomes. The financial shock and the associated decline in aggregate demand persistently tighten the lending threshold ν_t in (19). Like before, in the endogenous uncertainty model this leads to a persistent decline in lending activity.

Interestingly, while we feed in a shock to the supply side of the economy, the shock ends up lowering aggregate demand sufficiently to cause a demand-driven recession, with a positive labor wedge, similar to the logic in Guerrieri et al. (2022) and the evidence in Huber (2018).

5.3 Introducing Equity-Financed Firms

So far, all firms equally relied on bank credit in order to fund their projects, exposing their ability to operate to the beliefs of the financial market. We now explore the case in which some firms are equity-financed and do not need bank credit to fund the fixed cost $\phi_{i,t}$.[22] This allows them to always operate their preferred technology. To make it even starker, we assume away any information frictions for those firms as well. That is, equity-financed firms are able to observe $A_{i,t}$ at the end of each period, irrespective of the technology that was actually used in production. We explore the robustness of our mechanism to this extension by assuming that one half of all firms are equity-financed and thus never face any financial constraints.

Figure 9 shows the aggregate responses to a financial shock with the same magnitude as our baseline in Section 4. For comparison, we include the responses from the baseline model. Not surprisingly, the impact response is scaled down by the fraction of firms affected by the shock. Reassuringly, however, the responses are similarly persistent—if not more—compared

[22]Through other mechanisms, equity financing may also subject firms to the beliefs of the financial market, giving rise to a similar mechanism as the one in this paper. We explored this in a previous working paper version (Straub and Ulbricht, 2018).

with the baseline responses. Inasmuch as we do not have a strong prior about the magnitude of the exogenous shock, the two models hence behave very similarly in terms of measurable variables.

6 Quantitative Exploration

We next explore the quantitative relevance of endogenous uncertainty for businesses cycles. To do so, we build on the version of our model with capital (Section 5.1) and further extend it to allow for three additional standard shocks. We allow for shocks to total factor productivity (TFP) Z_t,

$$Y_{i,t} = Z_t \cdot \mathcal{A}_{i,t}^{\frac{1}{\xi-1}} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha},$$

shocks to the labor wedge τ_t^L, modifying the first order condition for labor supply from (24) to

$$\upsilon L_t^{1/\zeta} = \left(1 - \tau_t^L\right)\left(1 - \xi^{-1}\right)(1-\alpha)\, C_t^{-1} \boldsymbol{A}_t K_t^{\alpha} L_t^{-\alpha},$$

and shocks to the investment wedge τ_t^I, modifying the Euler equation from (23) to

$$\left(1 + \tau_{t-1}^I\right) C_t^{-1} = \mathbb{E}_t \left[\beta \left((1-\xi^{-1})\alpha \boldsymbol{A}_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + (1-\delta)\left(1+\tau_t^I\right)\right) C_{t+1}^{-1}\right].$$

We assume that all four shocks—the three just mentioned and the financial shock λ_t—are common knowledge and follow AR(1) processes. We estimate their standard deviations and persistences using Bayesian methods, following e.g. Smets and Wouters (2007) and Herbst and Schorfheide (2016), matching four aggregate time series. The first three are real GDP, real investment, nonfarm payrolls. As a simple measure of lending activity, we also match non-financial business lending transactions as percent of the trend in outstanding loans using new loans as a fraction of steady state outstanding loans in the model. All time series are de-trended by the Baxter and King (1999) filter. Prior distributions as well as the posterior modes and credible intervals are shown in Table 2.

We use the estimated model to investigate the role of endogenous uncertainty during historical business cycles. To do so, we “switch off” endogenous uncertainty by feeding in the estimated time series of shocks into the exogenous uncertainty version of the model. Table 3 compares time series moments across the two models. It computes the standard deviations and autocorrelations of output and hours, conditional on financial and non-financial shocks.

Non-financial shocks propagate very similarly with and without endogenous uncertainty. Both standard deviations and autocorrelations are virtually identical. It is financial shocks that propagate differently. With endogenous uncertainty, the standard deviations of output

Table 2: Priors and posteriors

Shock		Prior distribution	Posterior Mode	Posterior std. dev
TFP A_t	s.d.	Invgamma$(0.1, 2)$	0.368	(0.018)
	AR	Beta$(0.5, 0.1)$	0.218	(0.006)
L shock τ_t^L	s.d.	Invgamma$(0.1, 2)$	0.667	(0.032)
	AR	Beta$(0.5, 0.1)$	0.970	(0.015)
I shock τ_t^I	s.d.	Invgamma$(0.1, 2)$	0.086	(0.012)
	AR	Beta$(0.5, 0.1)$	0.861	(0.023)
Financial shock λ_t	s.d.	Invgamma$(0.1, 2)$	4.473	(0.218)
	AR	Beta$(0.5, 0.1)$	0.820	(0.028)

Note. Magnitudes are in percentage points.

and hours are about twice as large as they are without. The autocorrelations with lag 1 are high for both time series, but separate significantly more at higher lags, with the endogenous uncertainty model still having an autocorrelation of around 0.50 at lag 3 while the exogenous uncertainty model only has around 0.20.[23]

These results are at display in Figure 10, where we evaluate the contribution of endogenous uncertainty during the Great Recession. The black line is the data, which the endogenous uncertainty model matches exactly, with all four shocks active. The dashed red line is the exogenous uncertainty counterfactual, computed just as before, with the same estimated shocks, recovered from the endogenous uncertainty model. The plot clearly shows how, relative to 2008 Q1, output in the endogenous uncertainty model falls about twice as much and stays subdued significantly longer than in the exogenous uncertainty counterfactual.

Taken together, the results in this section suggest that endogenous uncertainty may indeed be an important propagator of financial shocks.

7 Firm-level vs Aggregate Uncertainty

A crucial assumption of our model is that agents are uncertain about firm-level productivity, implying that the feedback between credit and uncertainty operates entirely at the firm level. In this section, we explore the alternative case in which learning is about aggregate productivity instead. While the model has qualitatively similar predictions, we find that the scope for amplification due to aggregate uncertainty is quantitatively very small.

To explore learning about aggregate productivity, we include again a common aggregate

[23] We plot the full time series of output and hours with financial shocks in both models in Appendix D.4.

Table 3: Business cycle moments with and without endogenous uncertainty

	Financial shocks		Non-financial shocks	
	Endog. unc.	Exog. unc.	Endog. unc.	Exog. unc.
Output				
std. dev.	0.73	0.36	1.03	1.17
autocorr, lag 1	0.93	0.88	0.93	0.93
autocorr, lag 2	0.73	0.56	0.76	0.75
autocorr, lag 3	0.48	0.20	0.52	0.52
Hours				
std. dev.	0.48	0.24	1.06	1.02
autocorr, lag 1	0.93	0.88	0.95	0.95
autocorr, lag 2	0.73	0.56	0.82	0.81
autocorr, lag 3	0.47	0.19	0.63	0.62

Note. Columns 2 and 3 show the volatility and persistence of output and employment associated with financial shocks. Columns 4 and 5 show the volatility and persistence of output and employment associated with the three non-financial shocks (TFP, labor wedge and investment wedge).

TFP component Z_t,

$$Y_{i,t} = Z_t \cdot \mathcal{A}_{i,t}^{\frac{1}{\xi-1}} L_{i,t}.$$

We assume that Z_t follows an AR(1) process given by

$$Z_t = \rho_Z Z_{t-1} + (1-\rho)\bar{Z} + u_t,$$

where $\rho_Z \in [0,1]$ and the innovations u_t are i.i.d., normal, with zero mean and variance σ_u^2.[24]

To isolate the contribution of aggregate uncertainty, we assume that all firm-level productivities are perfectly observable at the end of the period, as in our exogenous uncertainty counterfactual.[25] The information set at the beginning of period t is given by

$$\mathcal{I}_t = \{\lambda_t, s_t\} \cup \{A_{i,t-1}\}_{i\in[0,1]} \cup \mathcal{I}_{t-1},$$

where s_t is a noisy signal of last period's aggregate output,

$$s_t = Y_{t-1} + \omega_t, \tag{26}$$

[24]Here we assume that $\bar{Z}$ is sufficiently large so $Z_t > 0$ with near certainty. Notice that for large $\bar{Z}$, the implied process for Z_t is virtually identical to the case where Z_t follows an AR(1) in logs. Assuming that Z_t follows an AR(1) in levels allows us to provide an exact characterization of the relevant uncertainty dynamics in closed form.

[25]This rules out the possibility that firm-level and aggregate uncertainty are complementary, which may potentially strengthen the role of aggregate uncertainty.

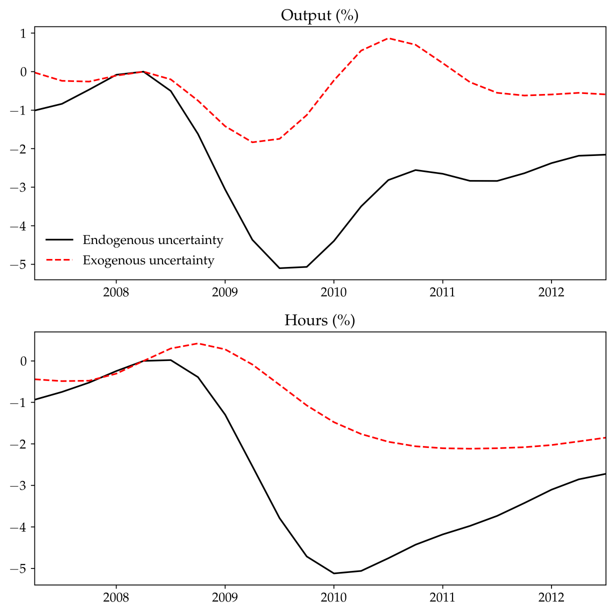

Figure 10: Contribution of endogenous uncertainty to the Great Financial Crisis

Note. This plot illustrates the role of endogenous uncertainty for the behavior of output and hours around the Great Financial Crisis. The black line is the data, which the endogenous uncertainty model matches exactly. The red line simulates the exogenous uncertainty model, subject to the same shock realizations as the endogenous uncertainty model. Both plots are normalized to 0 in 2008Q1.

with ω_t i.i.d., normal, with zero mean and variance σ_ω^2. To maximize the potential for aggregate uncertainty fluctuations, we assume that agents do not infer any information about Z_t from the cross-sectional distributions of prices, outputs, etc.

To characterize the uncertainty dynamics in this economy, define the aggregate input bundle in the economy as

$$X_t = \frac{Y_t}{Z_t} = \left(\int_0^1 \mathcal{A}_{i,t}^{\frac{1}{\xi}} L_{i,t}^{\frac{\xi-1}{\xi}} \, \mathrm{d}i \right)^{\frac{\xi}{\xi-1}},$$

and rewrite the signal (26) to get

$$s_t = Z_{t-1} X_{t-1} + \omega_t. \tag{27}$$

Observe that X_{t-1} is known by agents at time t. In this setting, aggregate financial shocks reduce the observed input X_{t-1}, which similar to van Nieuwerburgh and Veldkamp (2006) then reduces the signal precision of s_t, which in turn increases uncertainty about Z_t.

To evaluate the quantitative potential of the induced dynamics for aggregate uncertainty, suppose that the economy is in its stochastic steady state where $\mathbb{E}[Z_t|\mathcal{I}_t] = \bar{Z}$ and $\Sigma_t^Z \equiv$

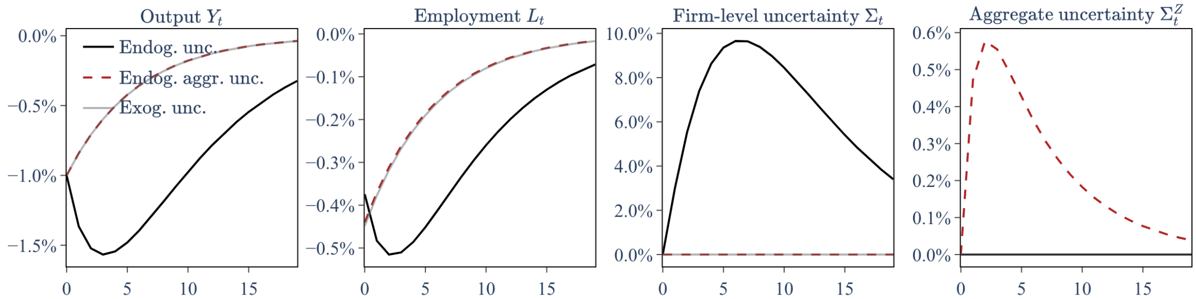

Figure 11: Endogenous uncertainty at the firm level vs. the aggregate level

$\text{Var}[Z_t|\mathcal{I}_t]$ converged to a constant. Now suppose the economy is hit by the same financial shock as in Section 4, whereas aggregate productivity and the noise shock remain at their steady state values ($Z_{t+s} = \bar{Z}$ and $\omega_{t+s} = 0$ for all $s \geq 0$). It then follows that agents' mean expectations remain unperturbed (i.e., $\mathbb{E}[Z_{t+s}|\mathcal{I}_{t+s}] = \bar{Z}$ for all s), and aggregate uncertainty evolves as follows

$$\Sigma_t^Z = \frac{\rho_Z^2 \Sigma_{t-1}^Z}{1 + (\sigma_\omega / X_{t-1})^{-2} \Sigma_{t-1}^Z} + \sigma_Z^2. \tag{28}$$

To maximize the potential impact of the aggregate uncertainty channel, we chose parameters ρ_Z, σ_Z and σ_ω so as to maximize the percentage increase in Σ_t^Z at the peak of the impulse response. Clearly, the response is maximized for $\rho_Z = 1$. Moreover, because any proportionate scaling of σ_Z and σ_ω also scales Σ_t^Z (and thus leaves the percentage response relative to steady state unchanged), it is sufficient to set the relative standard deviation σ_ω/σ_Z. We choose this ratio to maximize the peak response of uncertainty, which gives $\sigma_\omega/\sigma_Z = 2.052$.

Figure 11 shows the response in the aggregate-uncertainty model to a financial shock, alongside the responses with endogenous firm-level uncertainty and exogenous uncertainty. The exogenous uncertainty and endogenous aggregate uncertainty models are virtually identical. This is because even for a significant crisis with an output loss of 1% on impact, the ability to learn about *aggregate* productivity is only marginally affected. Panel (d) shows the maximized peak increase in aggregate uncertainty, which is below 0.6% (compared to an average increase in firm-level uncertainty of nearly 10% in our baseline model).

To see why this is the case, consider the signal (27). By design, X_t decreases by 1% on impact, decreasing the signal precision, $(X_{t-1}/\sigma_\omega)^2$, by only approximately 2%, severely limiting endogenous movements in aggregate uncertainty.

Would a larger crisis matter more? To explore this question, we scale up the financial shock and compute again the peak increase in uncertainty. Figure 12 shows the resulting link between the impact output loss and the peak increase in uncertainty using the above

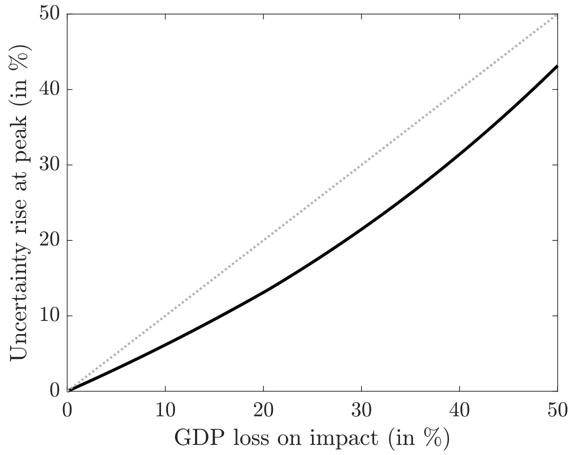

Figure 12: Peak increase in uncertainty by size of the financial shock in the aggregate uncertainty model

parameterization. It can be seen that for any magnitude of the shock, the peak increase in uncertainty is proportionately smaller than the corresponding loss in output. This is markedly different in our model with endogenous firm-level uncertainty. There, no matter how small the financial shock, it always results in some firms losing risky funding at the margin, starting the adverse credit–uncertainty spiral for those firms.

8 Concluding Remarks

We propose a theory of endogenous uncertainty and its interaction with firms' access to funds. In the model, firms rely on external funds to finance risky projects. When the returns to risky projects become too uncertain, firms are unable to obtain funds, resulting in a loss of information about the profitability of their projects. This further perpetuates funding problems. While present even in normal times, this feedback loop becomes especially powerful during financial crises, in which a temporary shock entails a prolonged economic downturn.

We have so far refrained from policy analysis in this paper. There are, however, several policy insights that merit further discussion. First, recapitalizing banks (or investors) is not an effective policy to restore lending in the model, once uncertainty has already increased. The critical friction that prolongs the crisis is an informational one and cannot easily be undone by transfers to banks, which would not stop the adverse feedback between uncertainty and bank lending, and in the extreme, may simply induce banks to hoard larger cash buffers. This, however, suggests a second policy action: direct transfers to firms as explored in Section 3.3. Even if the government has access to the same information as everyone else in the economy, providing transfers or cheap loans to inactive firms can crowd *in* lending in future periods. We view this as a fruitful avenue of further research.

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Endogenous Uncertainty and Credit Crunches

— Online Appendix —

Ludwig Straub Robert Ulbricht

A Evidence From Survey Data

At the core of our model is a two-way interaction between uncertainty and financial constraints, causing both variables to co-move. In this appendix section, we explore the extent to which this co-movement can be seen empirically, both in the micro-data and at the aggregate.

A.1 Data

Our dataset is a yearly panel of public US firms.

Proxies for uncertainty. Our proxy for uncertainty is based on forecasts about earnings per share (EPS) by financial analysts from the Institutional Brokers Estimate System (IBES). To make these forecasts comparable to our model, we follow Senga (2018) and transform EPS forecasts into forecasts about returns on assets (ROA). In our dataset, median productivity as measured by ROA is 3.7 percent (−9.5 percent at the 10th percentile, 13 percent at the 90th percentile). Let $\mu_{ij,t}^{\text{EPS}}$ denote analyst j's expectation about firm i's EPS at date t. Beliefs regarding returns on assets are computed as

$$\mu_{ij,t}^{\text{ROA}} = \mu_{ij,t}^{\text{EPS}} \times \frac{\text{number of outstanding shares}_{i,t}}{\text{total assets}_{i,t-1}}.$$

As our primary proxy for firm-level uncertainty, we look at the dispersion of forecast errors among analysts, defined by

$$\sigma_{i,t}^{\text{fce}} \equiv \text{sd}_j \left[\mu_{ij,t}^{\text{ROA}} - \text{ROA}_{i,t}\right].$$

Since $\text{ROA}_{i,t}$ is constant across all analysts j, $\sigma_{i,t}^{\text{fce}}$ can equivalently be interpreted as disagreement among forecasters.[A1]

[A1] It is worth noting that while there is a clear mapping in our model (equation (21)), it is not immediately clear that disagreement is also empirically a good measure of uncertainty. One concern stems from the fact that the precision σ_ψ^{-2} of the individual signal may change over time, too. Observe that this can go both ways, as the forecast dispersion is inverted-U shaped in the precision σ_ψ^{-2}. Absent any evidence that σ_ψ^{-2} moved one way or another, we interpret dispersion as measure of disagreement.

Proxies for financial constraints. For the purpose of measuring financial constraints, we follow the corporate finance literature and combine various balance sheet data to proxy for firms' access to funds. Our main measure is the "KZ-index" developed by Kaplan and Zingales (1997) and Lamont, Polk and Saá-Requejo (2001). Specifically, the "kz-score" of firm i at date t is given by

$$\mathrm{kz}_{i,t} = -1.001909 \times \frac{\mathrm{cashflow}_{i,t}}{k_{i,t-1}} + 0.2826389 \times Q_{i,t} + 3.139193 \times \frac{\mathrm{debt}_{i,t}}{\text{total capital}_{i,t}} - \\ - 39.3678 \times \frac{\mathrm{dividends}_{i,t}}{k_{i,t-1}} - 1.314759 \times \frac{\mathrm{cash}_{i,t}}{k_{i,t-1}},$$

where $\mathrm{cashflow}_{i,t}$ is the sum of COMPUSTAT items "income before extraordinary items" and "depreciation and amortization", $Q_{i,t}$ is ("market capitalization" + "total shareholder's equity" − "book value of common equity" − "deferred tax assets")/"total shareholder's equity", $\mathrm{debt}_{i,t}$ is "long-term debt" + "debt in current liabilities", total $\mathrm{capital}_{i,t}$ is "long-term debt" + "debt in current liabilities" + "stockholders equity", and $k_{i,t}$ is "total property, plants and equipment" (see the Appendix to Lamont, Polk and Saá-Requejo, 2001 for a listing of the corresponding COMPUSTAT items).

Intuitively, the KZ-score is a weighted combination of a firm's cash flow to total capital, its market to book ratio, its debt to capital, dividends to total capital, and cash holdings to capital. Firms with a higher $\mathrm{kz}_{i,t}$ score are more likely to be constrained.[A2]

Based on its $\mathrm{kz}_{i,t}$-score, we classify a firm as likely to be constrained if its current score is at or above the 95th percentile in a given calendar year. We also consider alternative proxies based on dividend payouts and debt to capital ratios, yielding similar results.

Timing and sample selection. Units of observation are defined by firm i and year t, where t refers to the year in which earnings are realized. Let $m_{i,t}$ denote the fiscal-year end month of firm i. All balance sheet data and realized earnings per share (EPS) for observation (i, t) are extracted at $m_{i,t}$. As in Senga (2018), we match each observation (i, t) with analysts' EPS-forecasts, $\mu^{\mathrm{EPS}}_{ij,t}$, extracted 8 months prior to $m_{i,t}$. That is, if in 2007, firm i's fiscal-year ends in March, then $\mu^{\mathrm{EPS}}_{ij,2007}$ would be extracted in July 2006.

From the original sample, we exclude all financial firms (SIC codes between 6000 and 6799) and firms in the electricity sector (SIC codes between 4900 and 4999). The resulting dataset ranges from 1976 to 2016 and covers, on average, 1979 firms per year. All variables are winsorized at the 1 percent level.

[A2]The weighting coefficients are based on an ordered logit regression relating those accounting variables to an explicit classification of firms into categories of financial constraints.

Table A.I: Financial constraints and uncertainty

	Data			
	(1)	(2)	(3)	(4)
Financially constrained	.081	.079	.079	.031
	(.012)	(.012)	(.012)	(.008)
Observations	47 342	47 342	47 335	46 141
Adj. R-sq.	.010	.023	.078	.709
Year × month FE	no	yes	yes	yes
Sector FE (4 digit)	no	no	yes	no
Firm FE	no	no	no	yes

Note. Standard errors clustered at the firm-level are in parenthesis.

A.2 Financial Constraints and Uncertainty

Cross-sectional evidence. To explore whether the predicted link between financial constraints and uncertainty is present in the data, we run a simple OLS regression of forecast-error dispersion $\sigma_{i,t}^{\text{fce}}$ on the KZ-based indicator. Table A.I reports the estimated coefficients, controlling for different combinations of fixed effects. The estimated effect is roughly constant over the first three specifications where we control for a combination of year, fiscal-end year month and 4-digit sector fixed effects. In all three specifications, the forecast-error dispersion is increased by about 0.08 for firms that are classified as financially constrained. Controlling for firm-level fixed effects, which arguably takes away a lot of variation in the financial constraint variable, the estimated difference between financially constrained and unconstrained firms is reduced to 0.031 but continues to be statistically significant. These results lend support to the model's predicted positive relationship between financial constraints and uncertainty.

Time series of forecast dispersion. Table A.I showed a strong significant relationship between forecast dispersion and financial constraints. What episodes in the data are responsible for this comovement? Figure A.I highlights that this is mainly driven by the recent financial crisis and previous crisis episodes. During the crises, uncertainty about constrained firms' fundamentals increases dramatically while uncertainty about unconstrained firms' fundamentals largely remains flat. Figure A.I also makes it obvious that forecast dispersion moves in the intuitive direction and increases significantly during the financial crisis.

A.3 Alternative Proxies for Financial Stress

In Table A.II we show additional results using two common alternative measures for financial constraints. The first is an indicator for whether dividend payouts are zero (Panel a), the

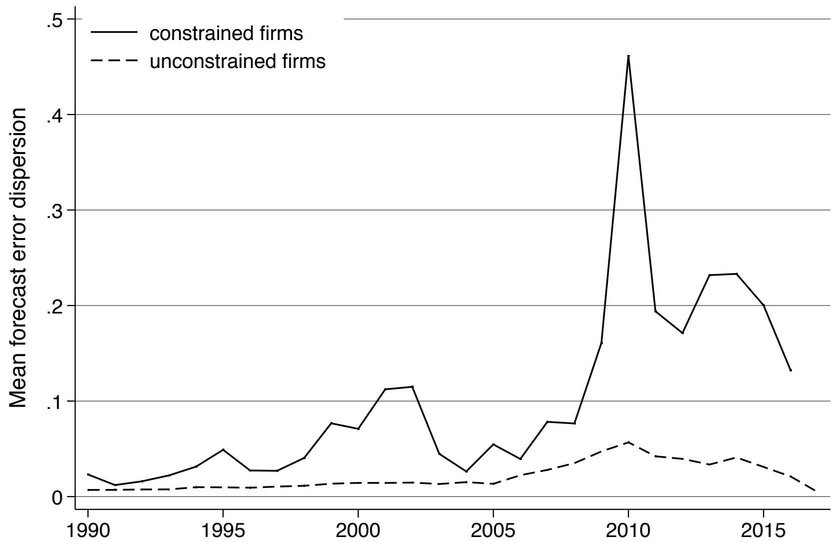


Figure A.I: Average forecast error dispersion (a proxy for uncertainty) of constrained and unconstrained firms.

Note. This figure shows the average forecast error dispersion among financially constrained and among financially unconstrained firms. Financially constrained firms are those whose current Kaplan and Zingales (1997) index lies in the top 5% of the distribution. Financially unconstrained firms are all other firms.

Table A.II: Alternative proxies for financial stress

	(1)	(2)	(3)	(4)
Panel a: Financial conditions measured by dividends				
Effect of constraint	.030	.026	.018	-.003
	(.002)	(.002)	(.003)	(.002)
Observations	58 737	58 737	58 735	57 215
Adj. R-sq.	0.009	0.022	0.072	0.700
Panel b: Financial conditions measured by leverage				
Effect of constraint	.016	.014	.015	.003
	(.005)	(.005)	(.004)	(.007)
Observations	58 641	58 641	58 639	57 124
Adj. R-sq.	0.000	0.015	0.070	0.706
Year × month FE	no	yes	yes	yes
Sector FE (4 digit)	no	no	yes	no
Firm FE	no	no	no	yes

Note: Standard errors clustered at the firm-level are in parenthesis.

second an indicator for whether the debt to capital ratio (which is a monotone function of leverage) is in the top 5% in a given year (Panel b).

The results are qualitatively similar to the ones in Table A.I. Quantitatively, the magnitudes in Table A.II are somewhat smaller compared to those in Table A.I. This is not surprising given that one may think of the KZ indicator as a (more or less) optimized indicator which already includes dividend payouts and leverage in its composition; and thus dividends and leverage are both relatively more noisy measures of financial constraints and therefore subject to greater attenuation bias.

B Mathematical Appendix

B.1 Proof of Proposition 1

Firm i at date t obtains a loan operating the risky technology if there exists an interest rate $r_{i,t} \geq \lambda_t$ such that

$$\Phi\left(\frac{\mu_{i,t} - \log\left(\left(1 + r_{i,t}\right)\phi\right) + \log\left(Y_t/w_t^{\xi-1}\right) - \log\left(\xi^{\xi}\left(\xi-1\right)^{1-\xi}\right)}{\sqrt{\Sigma_{i,t}}}\right) = \frac{1+\lambda_t}{1+r_{i,t}} \tag{A.1}$$

Define $x \equiv \frac{1+\lambda_t}{1+r_{it}} \in (0,1]$. Equation (A.1) is equivalent to there existing an $x \in (0,1]$ such that

$$\mu_{i,t} - \log\left(\left(1+\lambda_t\right)\phi\right) + \log\left(Y_t/w_t^{\xi-1}\right) - \log\left(\xi^{\xi}\left(\xi-1\right)^{1-\xi}\right) = \Phi^{-1}\left(x\right)\sqrt{\Sigma_{i,t}} - \log x$$

Observe that only the right hand side of this equation depends on x, and that it approaches infinity as $x \to 1$. Thus, the condition for a firm to be financed can be written as

$$\mu_{i,t} - \log\left(\left(1+\lambda_t\right)\phi\right) + \log\left(Y_t/w_t^{\xi-1}\right) - \log\left(\xi^{\xi}\left(\xi-1\right)^{1-\xi}\right) \geq \mathcal{V}\left(\Sigma_{i,t}\right)$$

with

$$\mathcal{V}\left(\Sigma_{i,t}\right) \equiv \min_{x\in(0,1]}\left\{\Phi^{-1}\left(x\right)\sqrt{\Sigma_{i,t}} - \log x\right\}.$$

This proves Proposition 1.

B.2 Properties of $\mathcal{V}(\Sigma)$

We prove a few properties of $\mathcal{V}(\Sigma)$ as well, which are stated in the text.

- $\mathcal{V}(0) = \min_{x\in(0,1]}\left\{-\log x\right\} = 0$.

- There is a unique minimizer in the definition of $\mathcal{V}(\Sigma)$. To see this, note that the FOC reads
$$\frac{1}{\phi\left(\Phi^{-1}(x)\right)}\sqrt{\Sigma} = \frac{1}{x}$$
where $\phi(\cdot)$ and $\Phi(\cdot)$ are the pdf and cdf of the standard normal distribution. Defining $z \equiv \Phi^{-1}(x) \in \mathbb{R}$, this can be rewritten as
$$\Phi(z)\sqrt{\Sigma} = \phi(z) \tag{A.2}$$
We claim that this is satisfied for a unique $z \in \mathbb{R}$. To see why, consider the derivatives of both sides
  - The derivative of the left hand side (LHS) is $\phi(z)\sqrt{\Sigma}$.
  - The derivative of the right hand side (RHS) is $\phi'(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}(-z)$.

  As $z \to -\infty$, both sides in (A.2) approach zero, but the LHS does so strictly faster, as it has a flatter, less positive derivative in that limit. Thus, $\Phi(z)\sqrt{\Sigma} < \phi(z)$ for sufficiently small z. Next, observe that the derivative of the LHS is strictly below the one of the RHS until $z = -\sqrt{\Sigma}$. After that, it is the other way around. This implies that there can be at most a single intersection. Existence of an intersection follows from the limit $z \to \infty$, where the LHS always strictly exceeds the RHS, in combination with the intermediate value theorem. We denote the unique minimizer in the definition of $\mathcal{V}(\Sigma)$ by $x^*(\Sigma)$, and define analogously $z^*(\Sigma) \equiv \phi^{-1}(x^*(\Sigma))$.

- We note from (A.2) that $z^*(\Sigma)$ is strictly decreasing in Σ, and thus $x^*(\Sigma)$ is strictly decreasing as well. It holds that $x^*(0) = 1$, $\lim_{\Sigma\to 0} z^*(\Sigma) = \infty$. Moreover, note that $z^*(2/\pi) = 0$, following straight from (A.2).

- $\mathcal{V}'(\Sigma) = \Phi^{-1}(x^*(\Sigma))\frac{1}{2\sqrt{\Sigma}} = \frac{z^*(\Sigma)}{2\sqrt{\Sigma}}$. Thus, $\mathcal{V}'(\Sigma)$ is strictly decreasing in Σ, with $\lim_{\Sigma\to 0}\mathcal{V}'(\Sigma) = \infty$. This shows that $\mathcal{V}'(\Sigma) > 0$ for $\Sigma < 2/\pi$.

B.3 Uniqueness of Equilibrium

Equation (18) can be rewritten as
$$\frac{\xi - 1}{\xi}\mathbf{A}_t = \upsilon\left(\frac{Y_t}{\mathbf{A}_t}\right)^{1/\zeta}\left(Y_t - \int_0^1 \phi_{i,t}\,\mathrm{d}i\right)$$

where $\boldsymbol{A}_t = F_t\left(\frac{Y_t}{w_t^{\xi-1}}\right)$ is an increasing function, and by (16), $w_t = (1-\xi^{-1})\boldsymbol{A}_t$. Combining these equations, we find a system of two equations and two unknowns, $\mathbf{A}_t$ and Y_t,

$$\frac{\xi-1}{\xi}\boldsymbol{A}_t = \upsilon\left(\frac{Y_t}{\boldsymbol{A}_t}\right)^{1/\zeta}\left(Y_t - \int_0^1 \phi_{i,t}\,\mathrm{d}i\right) \tag{A.3}$$

$$\boldsymbol{A}_t = F_t\left(\frac{Y_t}{((1-\xi^{-1})\boldsymbol{A}_t)^{\xi-1}}\right) \tag{A.4}$$

First, we observe that there always exists a solution to this system of equations. The reason is that (A.4) implies an increasing relationship between $\boldsymbol{A}_t$ and Y_t, which remains positive and bounded for $Y_t \to 0$ (when no firm is producing using the risky technology) and $Y_t \to \infty$ (when all firms are producing using the risky technology). This means that for $Y_t \to \int_0^1 \phi_{i,t}\,\mathrm{d}i$, the right hand side in (A.3) is smaller than the left hand side, and for $Y_t \to \infty$, the opposite is true. Thus, by the intermediate value theorem, there always exists a Y_t (and thus also an $\boldsymbol{A}_t$) that solve the system.

Next, we establish a condition for uniqueness. To do so, observe that, from (A.4) and the fact that $\boldsymbol{A}_t$ must be increasing in Y_t, we know the rate at which Y_t increases with $\boldsymbol{A}_t$ must be bounded above by the rate that would leave $\frac{Y_t}{((1-\xi^{-1})\boldsymbol{A}_t)^{\xi-1}}$ constant, that is,

$$\frac{d\log \boldsymbol{A}_t}{d\log Y_t} \leq \frac{1}{\xi-1}$$

Substituting this bound into (A.3) we find that indeed there can only be a single solution Y_t if

$$\frac{1}{\xi-1} < \frac{\frac{1}{\zeta} + \frac{Y}{Y-\int_0^1 \phi_{i,t}\,\mathrm{d}i}}{\frac{1}{\zeta}+1}$$

holds for any $Y_t > \int_0^1 \phi_{i,t}\,\mathrm{d}i$. This is satisfied if $\xi > 2$.

B.4 Proof of Proposition 2

From (19), the contour line is upward-sloping in $\mu_{i,t}$, implying that the upper arm of the Σ-locus ($\overline{\Sigma} = \sigma_\epsilon^2/(1-\rho^2)$) is overlapping with the lower arm ($\underline{\Sigma} = \sigma_\epsilon^2 < \overline{\Sigma}$) for some $\mu \in [\underline{\mu}, \overline{\mu}]$. Specifically, from (19), $\underline{\mu} = \mathcal{V}(\underline{\Sigma}) + \nu$ and $\overline{\mu} = \mathcal{V}(\overline{\Sigma}) + \nu$. Accordingly, both arms of the Σ-locus intersect the μ-locus ($\mu = \log \bar{A}$), if and only if

$$\mathcal{V}(\underline{\Sigma}) + \nu \leq \log \bar{A} \leq \mathcal{V}(\overline{\Sigma}) + \nu$$

or

$$\log \bar{A} - \mathcal{V}(\overline{\Sigma}) \leq \nu \leq \log \bar{A} - \mathcal{V}(\underline{\Sigma}).$$

Summarizing, there are two steady states whenever $\nu \in [\underline{\nu}, \overline{\nu}]$ with $\underline{\nu} = \log \bar{A} - \mathcal{V}(\sigma_\epsilon^2)$ and $\overline{\nu} = \log \bar{A} - \mathcal{V}(\sigma_\epsilon^2/(1-\rho^2))$. Otherwise, there is a unique steady state at either $\underline{\Sigma}$ (for $\nu < \underline{\nu}$) or at $\overline{\Sigma}$ (for $\nu > \overline{\nu}$).

B.5 Proof of Proposition 3

We derive the three equations in Proposition 3. We start with the firms' problem. Firms maximize

$$\Pi_{i,t} = p_{i,t} Y_{i,t} - r_t^K K_{i,t} - w_t L_{i,t}$$

subject to

$$Y_{i,t} = \mathcal{A}_{i,t}^{1/(\xi-1)} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha} \tag{A.5}$$

$$Y_{i,t} = Y_t p_{i,t}^{-\xi}$$

Observe that $\mathcal{A}_{i,t}$ is the only object that is not a choice object and different across firms i. We conjecture that $K_{i,t}$ and $L_{i,t}$ scale with $\mathcal{A}_{i,t}$; that $p_{i,t}$ scales with $\mathcal{A}_{i,t}^{-1/(\xi-1)}$; and that $Y_{i,t}$ scales with $\mathcal{A}_{i,t}^{\xi/(\xi-1)}$. To verify this conjecture, observe that: (a) profits can be rewritten as

$$\Pi_{i,t} = \mathcal{A}_{i,t} \cdot \left[\frac{p_{i,t}}{\mathcal{A}_{i,t}^{-1/(\xi-1)}} \frac{Y_{i,t}}{\mathcal{A}_{i,t}^{\xi/(\xi-1)}} - r_t^K \frac{K_{i,t}}{\mathcal{A}_{i,t}} - w_t \frac{L_{i,t}}{\mathcal{A}_{i,t}} \right]$$

where the term in brackets only depends on scale-free objects; (b) the production function can be rewritten in scale free terms as well,

$$\frac{Y_{i,t}}{\mathcal{A}_{i,t}^{\xi/(\xi-1)}} = \left(\frac{K_{i,t}}{\mathcal{A}_{i,t}} \right)^{\alpha} \left(\frac{L_{i,t}}{\mathcal{A}_{i,t}} \right)^{1-\alpha}$$

and so can the demand for i's goods,

$$\frac{Y_{i,t}}{\mathcal{A}_{i,t}^{\xi/(\xi-1)}} = Y_t \left(\frac{p_{i,t}}{\mathcal{A}_{i,t}^{-1/(\xi-1)}} \right)^{-\xi}.$$

This establishes that the conjecture was correct.

From market clearing, we know that

$$\int_0^1 K_{i,t} \, \mathrm{d}i = K_t$$

Since $K_{i,t}$ is proportional to $\mathcal{A}_{i,t}$, this implies that

$$K_{i,t} = \frac{\mathcal{A}_{i,t}}{\boldsymbol{A}_t^{\xi-1}} K_t \tag{A.6}$$

where $\boldsymbol{A}_t$ is defined in (17). Similarly,

$$L_{i,t} = \frac{\mathcal{A}_{i,t}}{\boldsymbol{A}_t^{\xi-1}} L_t. \tag{A.7}$$

Using (A.5), (A.6) and (A.7) aggregate output is then given by

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\xi-1}{\xi}} \, \mathrm{d}i \right)^{\frac{\xi}{\xi-1}} = \boldsymbol{A}_t K_t^{\alpha} L_t^{1-\alpha} \tag{A.8}$$

with

$$r_t^K = \alpha(1-\xi^{-1}) \frac{p_{i,t} Y_{i,t}}{K_{i,t}} = \alpha(1-\xi^{-1}) \frac{Y_t}{K_t} = \alpha(1-\xi^{-1}) \boldsymbol{A}_t K_t^{\alpha-1} L_t^{1-\alpha} \tag{A.9}$$

and similarly,

$$w_t = (1-\alpha)\,(1-\xi^{-1}) \boldsymbol{A}_t K_t^{\alpha} L_t^{-\alpha}. \tag{A.10}$$

The Euler equation from households is standard, and given by

$$C_t^{-1} = \mathbb{E}_t \left[\beta \left(1 + r_{t+1}^K - \delta \right) C_{t+1}^{-1} \right]$$

Substituting in (A.9) yields (23). The optimality condition for labor is standard and given by

$$\upsilon L_t^{1/\zeta} = C_t^{-1} w_t.$$

Substituting in (A.10) gives (24). Finally, the resource constraint (25) follows from (A.8).

C Solving the Model

We solve all variants of our model in the sequence space, using the tools provided by Auclert et al. (2021). For brevity we only describe how to solve the model with investment; the model without investment is simply a special case with zero capital share and no depreciation; the model with nominal rigidities is solved in a very similar fashion to the model here. Using the language in that paper, the baseline model consists of four "blocks":

1. firm block [heterogeneous-agent block]: The firm block maps real marginal input cost mc_t, the financial shock λ_t, aggregate TFP Z_t, aggregate demand Y_t^d into

- aggregate output Y_t
- efficiency wedge $\boldsymbol{A}_t$
- total operating costs G_t

We compute these objects by iterating over the distribution of firms in belief space, $g_t(\mu, \Sigma, d)$ where $d \in \{0, 1\}$ is an indicator for whether a firm is in default or not. Each period goes through the following stages:

- We start with the previous end-of-period distribution $g_t^{(0)} \equiv g_{t-1}$.
- We move a random fraction η of defaulted firms back into no-default,

$$
\begin{aligned}
g_t^{(1)}(\mu, \Sigma, 0) &= g_t^{(0)}(\mu, \Sigma, 0) + \eta g_t^{(0)}(\mu, \Sigma, 1) \\
g_t^{(1)}(\mu, \Sigma, 1) &= g_t^{(0)}(\mu, \Sigma, 1) - \eta g_t^{(0)}(\mu, \Sigma, 1)
\end{aligned}
$$

- We label by Σ_k the uncertainty associated with not having received a signal for k periods,

$$
\begin{aligned}
\Sigma_0 &= 0 \\
\Sigma_{k+1} &= \rho^2 \Sigma_k^2 + \sigma_\epsilon^2 \qquad k \geq 0
\end{aligned}
$$

- We evolve beliefs to be over $\log A_{i,t}$ instead of $\log A_{i,t-1}$,

$$
g_t^{(2)}(\mu, \Sigma_{k+1}, d) = (1-\theta) \int_{\tilde{\mu}} g_t^{(1)}(\tilde{\mu}, \Sigma_k, d) \; \mathrm{d}\Phi\left(\frac{\mu - \rho\tilde{\mu} - (1-\rho)\log \bar{A}}{\sigma_\epsilon^2}\right), \qquad k \geq 1
$$

$$
g_t^{(2)}(\mu, \Sigma_1, d) = \theta \sum_{k \geq 1} \int_{\tilde{\mu}} g_t^{(1)}(\tilde{\mu}, \Sigma_k, d) \; \mathrm{d}\Phi\left(\frac{\mu - \rho\tilde{\mu} - (1-\rho)\log \bar{A}}{\sigma_\epsilon^2}\right)
$$

$$
g_t^{(2)}(\mu, \Sigma_0, d) = 0
$$

- We update beliefs based on the funding condition (13) for firms not in default, $d = 0$,
  - for firms that do not receive funding, for any $k \geq 1$, nothing changes,

$$
g_t^{(3)}(\mu, \Sigma_k, 0 | \mu_-, \Sigma_-) = 1_{\{\mu=\mu_-, \Sigma_k=\Sigma_-\}} 1_{\{\mu < \mathcal{V}(\Sigma_k)+\nu_t\}} g_t^{(2)}(\mu, \Sigma_k, 0)
$$

– while uncertainty for firms that receive funding drops to $\Sigma_0 = 0$,

$$g_t^{(3)}(\mu, \Sigma_0, 0|\mu_-, \Sigma_-) = \Phi\left(\frac{\mu - \mu_-}{\Sigma_-}\right) \sum_{k \geq 0} 1_{\{\mu_- \geq \mathcal{V}(\Sigma_-) + \nu_t\}} g_t^{(2)}(\mu_-, \Sigma_-, 0)$$

– nothing happens to firms in default

$$g_t^{(3)}(\mu, \Sigma, 1|\mu_-, \Sigma_-) = 1_{\{\mu=\mu_-, \Sigma=\Sigma_-\}} g_t^{(2)}(\mu, \Sigma, 1)$$

- We keep track of beliefs at the stage of funding as μ_-, Σ_- in this notation.
- We compute the firm-specific lending rate $r_t(\mu, \Sigma)$ as the smallest solution of (12), rewritten for the more general case here,

$$-\Phi^{-1}\left(\frac{1 + \lambda_t}{1 + r_t(\mu, \Sigma)}\right)\sqrt{\Sigma} + \mu - \log\big(1 + r_t(\mu, \Sigma)\big) + \omega_t = 0$$

for some aggregate composite

$$\omega_t \equiv (\xi - 1)\log Z_t + \log\left(Y_t/\mathrm{mc}_t^{\xi-1}\right) - \log\left(\xi^\xi\,(\xi - 1)^{1-\xi}\,\phi\right).$$

If no solution exists, we set $r_t(\mu, \Sigma) = \infty$.

- Finally, some firms default according to (11),

$$g_t(\mu, \Sigma_0, 1) = \sum_k \int 1_{\{\mu < \log(1 + r_t(\mu_-, \Sigma_k)) - \omega_t\}} g_t^{(3)}(\mu, \Sigma_0, 0|\mu_-, \Sigma_k)\,\mathrm{d}\mu_-$$

and the rest does not,

$$g_t(\mu, \Sigma_0, 0) = \sum_k \int 1_{\{\mu \geq \log(1 + r_t(\mu_-, \Sigma_k)) - \omega_t\}} g_t^{(3)}(\mu, \Sigma_0, 0|\mu_-, \Sigma_k)\,\mathrm{d}\mu_-$$

$$g_t(\mu, \Sigma_k, d) = g_t^{(3)}(\mu, \Sigma_k, d|\mu, \Sigma_k) \qquad \text{for } k \geq 1$$

We then compute:

- Firm level output

$$y_t^{\text{risky}}(\mu) = \left(\frac{\xi - 1}{\xi}\right)^\xi Z_t^\xi e^{\frac{\xi}{\xi-1}\mu} \frac{Y_t^d}{\mathrm{mc}_t^\xi}$$

$$y_t^{\text{base}} = \left(\frac{\xi - 1}{\xi}\right)^\xi Z_t^\xi \left(\tilde{A}\right)^{\xi/(\xi-1)} \frac{Y_t^d}{\mathrm{mc}_t^\xi}$$

- Aggregating, we find aggregate output from

$$Y_t^{1-1/\xi} = \int \left(y_t^{\text{risky}}(\mu) \right)^{1-1/\xi} g_t(\mu, \Sigma_0, 0) d\mu + \left(y_t^{\text{base}} \right)^{1-1/\xi} \cdot \left(1 - \int g_t(\mu, \Sigma_0, 0) d\mu \right),$$

the efficiency wedge from

$$\mathbf{A}_t^{\xi-1} = \int e^{\mu} g_t(\mu, \Sigma_0, 0) d\mu + \tilde{A} \cdot \left(1 - \int g_t(\mu, \Sigma_0, 0) d\mu \right),$$

and total operating cost G_t from

$$G_t = \phi \int g_t(\mu, \Sigma_0, 0) d\mu + \tilde{\phi} \left(1 - \int g_t(\mu, \Sigma_0, 0) d\mu \right)$$

We compute the Jacobian of this block as in the "forward iteration" step in Auclert et al. (2021).

2. Value added block [simple block]: The value added block maps the aggregate sequences for output Y_t, real marginal input cost mc_t, capital K_t, the efficiency wedge $\boldsymbol{A}_t$, aggregate TFP Z_t, and the investment wedge τ_t^I into

   - labor demand $L_t^d = \left(\frac{Y_t}{Z_t \mathbf{A}_t K_{t-1}^{\alpha}} \right)^{\frac{1}{1-\alpha}}$
   - real wage $w_t = (1-\alpha) \frac{\text{mc}_t}{Z_t \mathbf{A}_t} \frac{Y_t}{L_t^d}$
   - return on capital $R_t = \alpha \frac{\text{mc}_t}{Z_t \mathbf{A}_t} \frac{Y_t}{K_{t-1}} + (1-\delta) \left(1 + \tau_t^I \right)$
   - real interest rate $r_t = \frac{R_t}{1+\tau_{t-1}^I} - 1$
   - investment $I_t = K_t - (1-\delta) K_{t-1}$

3. Household block [simple block]: The household block maps the four aggregate sequences for output Y_t, real wages w_t, investment I_t, total operating cost G_t, and the labor wedge τ_t^L into

   - consumption $C_t = Y_t - I_t - G_t$
   - real interest rate $r_t = \beta^{-1} C_{t+1}/C_t - 1$
   - labor supply $L_t^s = \left(\left(1 - \tau_t^L \right) \frac{w_t}{\upsilon C_t} \right)^{\zeta}$

4. Market clearing block [simple block]: The market clearing block maps labor demand and supply L_t^d, L_t^s, the real rate r_t, consumption C_t, output and demand Y_t, Y_t^d into

   - labor market clearing: labor $\text{mkt}_t = L_t^d - L_t^s$

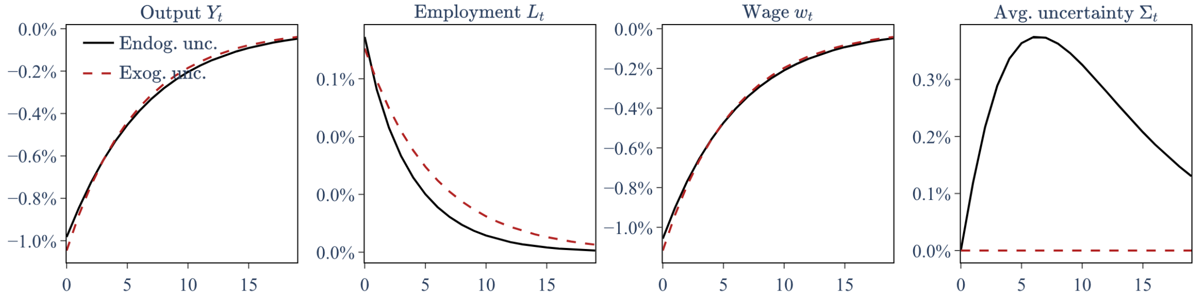


Figure A.II: Response to an aggregate productivity shock

- the Euler condition: $\text{euler}_t = 1 + r_{t+1} - \beta^{-1}\frac{C_{t+1}}{C_t}$
- aggregate output condition: $\text{output mkt}_t \equiv Y_t - Y_t^d$

The three unknowns of this model are real marginal input cost mc_t, capital K_t, and aggregate demand Y_t^d. The three targets are labor mkt_t, euler_t, and output mkt_t. The four shocks are the financial shock λ_t; TFP Z_t; the investment wedge τ_t^I; and the labor wedge τ_t^L.

D Additional Results

D.1 Aggregate Productivity Shocks

Our focus in this paper is on shocks to the financial sector. One may wonder, however, whether our model with its financial and information frictions also fundamentally alters the response to aggregate productivity shocks. To do so, suppose production is subject to a common, fully known, aggregate productivity shock Z_t,

$$Y_{i,t} = Z_t \cdot \mathcal{A}_{i,t}^{\frac{1}{\xi-1}} L_{i,t}.$$

Figure A.II shows that the endogenous and exogenous uncertainty models behave nearly identically in response to the aggregate productivity shock.[A3] This is because an aggregate productivity shock does not shift ν_t nearly as much as the financial shock, as the response of average uncertainty in Figure A.II shows.

D.2 Robustness to the Fraction of Financially Constrained Firms

In our calibration in Section 4.1, we worked with a parameterization that targeted a steady state share of 25% of constrained firms that do not have access to funding for the risky

[A3] We use the same half-life of 4 quarters as above and normalize the impact response of output to -1%.

Table A.III: Parameters for robustness exercise

Parameter	β	ζ	ξ	υ	$\bar{A}$	$\tilde{A}$	ρ	σ_ϵ	ϕ	η	θ
Baseline value	0.99	2.000	5.000	0.913	1.041	0.563	0.944	0.073	0.139	0.350	0.117
20% constrained	0.99	2.000	5.000	0.910	1.028	0.563	0.944	0.073	0.133	0.350	0.110
15% constrained	0.99	2.000	5.000	0.905	1.015	0.563	0.944	0.073	0.125	0.350	0.082

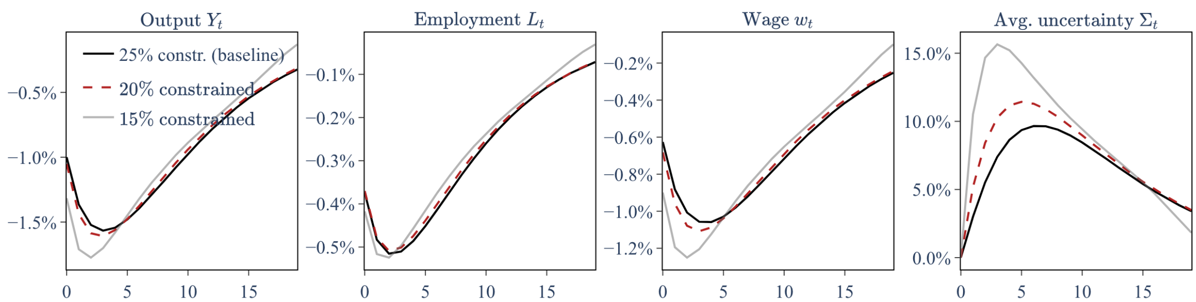


Figure A.III: Robustness with respect to the share of constrained firms

Note. All panels are denominated in percentage deviations from the steady state. Solid black lines are the baseline model responses; dashed red lines correspond to the recalibrated model with a 20% share of constrained firms; gray lines correspond to the recalibrated model with a 15% share of constrained firms.

technology. In this section, we explore robustness to shares of constrained firms of 15% and 20%. Table A.III shows the recalibrated parameters.

In Figure A.III we compare the impulse responses in our baseline model with those in the two recalibrated models. The patterns are broadly similar across models, although amplification and persistence are weaker with smaller shares of constrained firms.

D.3 Alternative exogenous uncertainty benchmarks

In Figure A.IV, we show three different way of specifying the exogenous uncertainty benchmark used in Figure 5 and after. Our baseline assumption is to calibrate ϕ to hit the same impact response of output as the endogenous uncertainty model (black solid). We could similarly also use the same parameters as in the endogenous uncertainty model (red dashed), or assume that, each period, the same number of firms become constrained (gray solid). Conditional on the same output impact response, all three versions have the same subsequent evolution of output, so predictions for persistence are identical. The paths of employment, wages, and consumption are also very similar across the three models. The parameters of these alternative models are shown in Table A.IV.[A4]

[A4]Observe that in the "same parameters" model, $\overline{A}$ and ϕ are rescaled by the same amount so that the lending threshold is unchanged relative to the endogenous uncertainty model. $\overline{A}$ is used to hit the normalization $Y = 1$ and υ is used to hit the normalization $N = 1$.

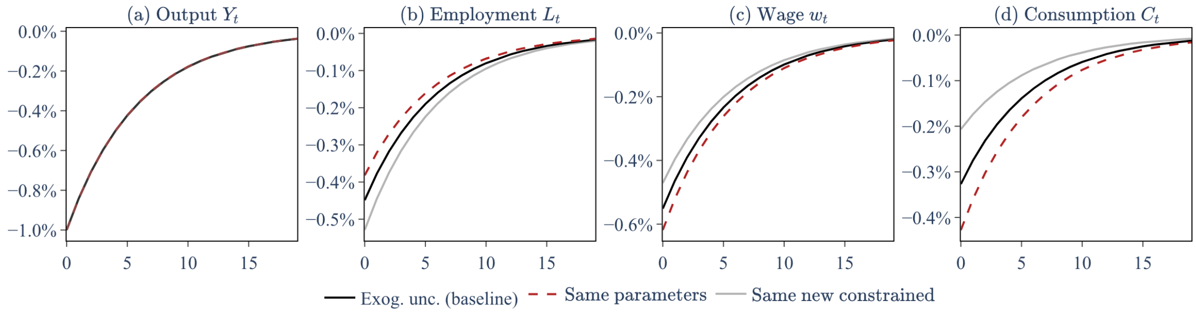


Figure A.IV: Comparing different ways of defining the exogenous uncertainty benchmark

Note. Panels compare different ways of defining the exogenous uncertainty benchmark.

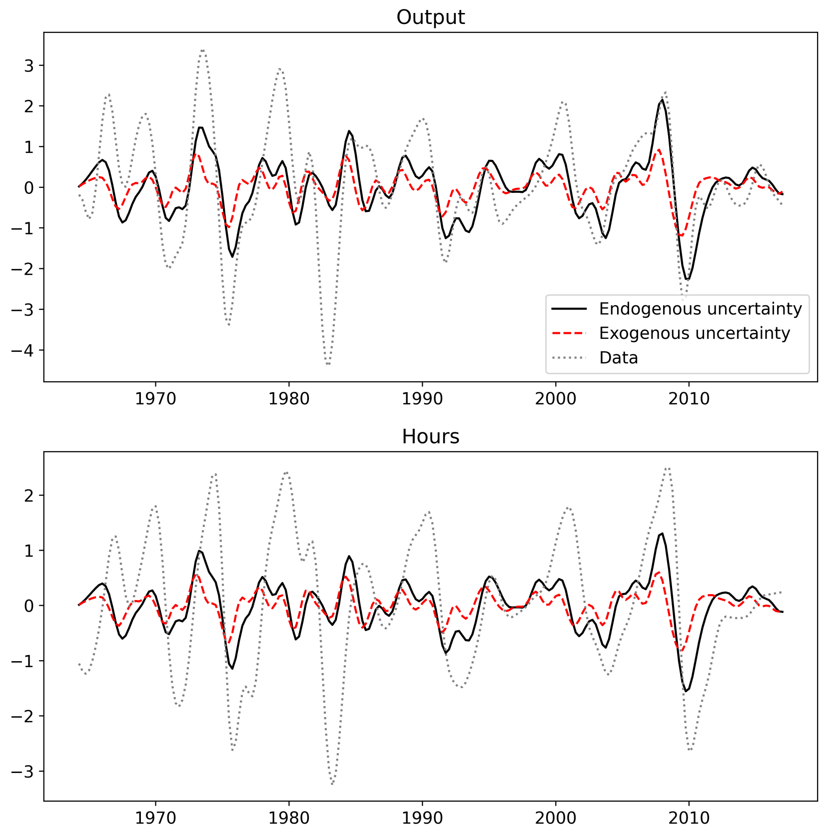
Table A.IV: Calibrated parameters of alternative models

Exog. uncertainty model	β	ζ	ξ	υ	$\bar{A}$	$\tilde{A}/\bar{A}$	ρ	σ_ϵ	ϕ	η	θ
Baseline	0.99	2.000	5.000	0.907	0.983	0.563	0.944	0.073	0.121	0.350	1.000
Same parameters	0.99	2.000	5.000	0.915	0.996	0.563	0.944	0.073	0.133	0.350	1.000
Same new constrained	0.99	2.000	5.000	0.899	0.978	0.563	0.944	0.073	0.112	0.350	1.000

D.4 Role of endogenous uncertainty for output and hours

Figure A.V illustrates the role of endogenous uncertainty for the historical paths of output and hours. To construct it, we start from the estimated endogenous uncertainty model, which is designed to match the data (dotted line). We plot the contribution of financial shocks (black solid), where endogenous uncertainty matters most. The exogenous uncertainty benchmark (red dashed) is obtained by feeding the exact pattern of historical financial shocks estimated for the endogenous uncertainty model into the model with exogenous uncertainty.

Figure A.V: Role of endogenous uncertainty for contribution of financial shocks to the business cycle



Note. This plot shows the estimated historical contribution of financial shocks to output and hours in the endogenous uncertainty model (black). The red, dashed line is the historical path of output and hours in the exogenous uncertainty benchmark, when it is subject to the same set of historical shocks as the endogenous uncertainty model. Dotted is the data, which differs from the black line due to the presence of other shocks.