Collective Behavior with Information Asymmetry

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Abstract

We propose a new method for identifying bargaining power in collective household models, based on information asymmetry. Our model allows household members to exploit an information advantage for bargaining. We formulate the household's decision process under partial information disclosure using a Bayesian persuasion framework. We use this structure to point identify utility and bargaining power, which would not be identified under symmetric information. We illustrate these results by showing that our model can explain known empirical outcomes regarding child educational investment and development in Chinese households where one parent is a migrant.

Key words: Collective model; Information asymmetry; Bargaining power; Bayesian persuasion; Left-behind children

JEL Codes: D11; D13; D82; J13

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1 Introduction

The collective model, pioneered by Becker (1981) and Chiappori (1988, 1992), is widely used for analyzing household behavior. We address two issues in this literature. First, there is a scarcity of research that incorporates information asymmetry into formal models of household behavior, even though many experimental studies document significant instances and implications of asymmetric information in household decisions. For example, income hiding by spouses is well documented in the development literature. See, e.g., Castilla (2019) and references therein. Other examples include Ashraf (2009), Castilla and Walker (2013), Doepke and Tertilt (2016), Apedo-Amah et al. (2020), and Ashraf et al. (2022). Second, when the model implies Pareto-efficient decisions, the Pareto weights, a measure of intra-household bargaining power,[1] are often difficult to identify. As emphasized in Chiappori and Mazzocco (2017), integrating asymmetric information and achieving identification results are crucial for policy design and evaluation.

A famous result in the efficient collective household model literature is the non-identification[2] of utility and Pareto weights from continuous demand data, unless one imposes strong behavioral restrictions. See, e.g., Chiappori and Ekeland (2009). We show that a similar non-identification holds for discrete household decisions such as binary choice, but surprisingly, identification of utility and Pareto weights becomes possible when there is asymmetric information among household members.

We illustrate these results by showing that asymmetric information (along with taste variation) can explain known empirical outcomes regarding child educational investment and development in Chinese households where one parent is a migrant.

Standard Samuelson-Houthakker revealed preference theory says that the (ordinal) utility function of a single utility-maximizing consumer can be identified from that consumer's observable continuous demand functions. Chiappori and Ekeland (2009), among others, show that this identification does not extend to efficient collective households: the utility and relative bargaining power of household members cannot be identified just from

[1]In two-person households, efficiency guarantees that the household behaves as if it were maximizing a weighted average of the utility functions of the two household members. The weights on these utility functions, known as Pareto weights, are interpreted as a measure of relative bargaining power of the two household members.

[2]Throughout this paper, when we refer to identification, we mean point identification. In contrast, set identification in collective household models is possible just from household demand functions. See, e.g., Cherchye et al. (2015) and Cherchye et al. (2017).

a household's observable demand functions. Additional behavioral or data assumptions are required. Examples of such assumptions that have been proposed to obtain collective model identification include preference similarity restrictions across people, strong functional form restrictions, or assuming that some goods are known to be assignable—i.e., consumed by only one household member.[3,4]

This collective household literature starts from knowledge that suffices to identify a single person's preferences over continuous goods, shows that this knowledge does not identify collective household model utilities and bargaining power, and then adds additional model assumptions that make collective household model identification possible.

In this paper, we do the same for discrete decision models, such as the choice of whether to purchase a single product or not. We start from assumptions that suffice to identify the preferences of a single person (e.g., a logit or probit model). We show that this knowledge is not enough to identify collective household utilities and bargaining power, and then provide a new modeling assumption, asymmetric information, that suffices for identification. Though one could argue that, rather than adding an assumption, we are relaxing the assumption of symmetric information.

In a logit or probit model, an individual's utility function is $v + e$ if he chooses action a_1 and zero otherwise. Here v, which generally would be a function of covariates, is the individual's deterministic utility level from choosing a_1, and e is a state-specific random component that is observed by the individual. Maximizing utility, the individual chooses a_1 if e exceeds a cutoff c^* that suffices to make utility $v + e$ positive, so $c^* = -v$. The researcher is assumed to observe the probability p that the individual chooses a_1. This p is identified either by observing the same utility-maximizing individual making choices many times, or by observing the choices of many individuals who are assumed to have similar preferences. Observing p here is the analog to observing quantity demand functions in

[3]Examples of collecting detailed consumption data for individual household members including the fraction of shared goods that each individual consumes are Cherchye et al. (2012) and Menon et al. (2012). Papers that attain identification by imposing behavioral or functional form restrictions on preferences of individuals within or across households include Lewbel and Pendakur (2008), Lise and Seitz (2011), Bargain and Donni (2012), Browning et al. (2013), and Dunbar et al. (2013). The latter paper, along with Lechene et al. (2022) assume assignable goods.

[4]The use of distribution factors (covariates that affect bargaining but not tastes) has also been proposed for identification. The effects of changing a distribution factor on changes in bargaining power are identified, but by themselves distribution factors cannot identify the level of bargaining power. See, e.g., Browning et al. (1994), Fong and Zhang (2001), Chiappori et al. (2002), Blundell et al. (2005), Chau et al. (2007), and Bourguignon et al. (2009).

standard revealed preference theory. Assume that G, the cumulative distribution function of e, is known to the researcher (logistic in the case of logit models, or standard normal in the case of probit models). This standard assumption then suffices to identify the utility level v from $p = 1 - G(-v)$.

In our extension of this model to the collective household, a husband and wife have utility $v^h + e$ and $v^w + e$, respectively, from choosing a household-level action a_1 and zero otherwise. Under symmetric information, where both spouses observe the realization of e simultaneously, efficiency again results in the household choosing to take action a_1 if e exceeds a cutoff c^*, but now c^* is determined by $\left(v^h + c^*\right)\lambda^h + v^w + c^* = 0$ with λ^h being the Pareto weight that (relative to one) defines the husband's relative bargaining power. In this model, knowing p (the probability that the household chooses a_1) and G is not sufficient to identify any of the parameters v^h, v^w, or λ^h.[5] We prove this non-identification following Proposition 1 below.

We then consider an asymmetric information scenario where one household member, say the wife, observes e, and the other does not. We formulate the household's decision process under partial information disclosure using the Bayesian persuasion framework (Kamenica and Gentzkow, 2011).[6] Using this framework we obtain the household's equilibrium condition and solve the model.[7] The result is that there will still be a cutoff c^* such that the household chooses a_1 if e exceeds c^*, but c^* is a more complicated function than before.

Depending on the relative values of the above parameters, the wife will either choose to fully reveal the value of e to her husband or not. If she does reveal e, the husband's bargaining power will be given by λ^h above. But if she chooses not to reveal e, we show

[5] In discrete choice models, the deterministic utility values v^h and v^w depend on the normalization of the variance of the random utility component e, given that overall scale of utility is irrelevant (Train, 2009). Nevertheless, the point identification of v under the specified assumptions of e becomes crucial, as it facilitates market aggregation, welfare analyses, and counterfactual policy evaluations with collective household decision-making. In Section 3's numerical analysis, we investigate how our findings are sensitive to varying assumptions about the distribution of e.

[6] Prior studies like Kamenica (2019) typically either consider multiple players learning the information or consider multiple players being uninformed. Our analysis extends to encompass both scenarios.

[7] The wife can either fully disclose e to her husband or not, depending on whichever choice will yield her higher utility. Full information disclosure is equivalent to no information asymmetry, as both spouses then become informed upon the realization of e. The alternative to full disclosure is the wife devising a recommendation strategy before e is realized. Following the realization, she recommends a choice to her husband in accordance with her recommendation strategy. The husband, upon receiving his wife's recommendation, updates his belief regarding the information and then determines whether to accept. The resulting equilibrium describes the household's behavior.

that the husband will have a different Pareto weight λ^{h*} that is a function of v^h, v^w, and c^*. The wife maximizes her own utility by making the information revealing choice that yields the lower of λ^h and λ^{h*}. This variation in Pareto weights resulting from the wife's information choice is what allows us to potentially identify the parameters v^h, v^w, and λ^h. Examining the equilibrium solution in detail also reveals that the husband's relative bargaining power decreases the larger is the variance of e, and decreases the closer v^h is to v^w.

To illustrate our results, we consider a child investment decision in households with a migrant parent. The parent staying at home with the child likely possesses better information regarding the child's talents and development potential, while the migrating parent is less informed. Parents decide between making a high-level and a low-level educational investment in the child, and derive utility from both consumption and child quality. Child quality is influenced by the level of investment and by the child's inherent abilities, with a more gifted child exhibiting a larger quality enhancement from the high investment over the low one. Parental utility from forgone consumption due to investment is deterministic and differs between husband and wife. We assume that both parents derive a common (random) utility from child quality, but the wife is more inclined than the husband to sacrifice current consumption for child investment.[8] This is equivalent to e being the same for both spouses, but $v^w > v^h$. If one spouse stays at home while the other migrates, the spouse who stays at home is assumed to observe e.

With $v^w > v^h$, our model implies that, ceteris paribus, the investment in left-behind children will be higher if the wife stays behind and the husband migrates than vice versa. This is consistent with, and so could help explain, the established empirical regularity that left-behind children often fare better when staying with mothers than with fathers (e.g., Zhang et al., 2014).

Finally, we extend our model to situations with multiple choices and then to multiple players. Our conclusions about the optimal decision and bargaining power remain similar to those in the two-choice, two-player case, though they require some additional assumptions about the structure of players' utility. We show that identification can still be achieved in these extensions, based on independent moment conditions that arise from

[8]One reason could be evolutionary: the husband has a longer reproductive horizon, and so may wish to spend less on a current child and more on potential future children (and partners) (Trivers, 2017).

varying relationships among players' preferences and/or their information access. The number of moment conditions required depends on the number of choices and players.

In the sense that information asymmetry affects spouses' bargaining power without affecting their preferences or the budget constraint, this asymmetry is an example of a distribution factor. However, in contrast to standard distribution factors,[9] information asymmetry allows us to identify the level of bargaining power in our model (and not just how power changes as a function of the distribution factor). Our results suggest that information asymmetry may be a generally useful tool for obtaining identification in collective household models.

The paper is organized as follows. Section 2 sets out a collective model with information asymmetry. Section 3 applies the model to analyze investment decisions and child development in households with a migrant parent. Section 4 extends the model to incorporate multiple choices and multiple players. Section 5 concludes.

2 A collective model with information asymmetry

In this section, we present a two-player collective binary choice model. We first analyze the model under symmetric full information, and then consider asymmetric information.

2.1 State-specific utilities and collective decisions

Consider a household with a husband and a wife, $m \in \{h, w\}$, that faces a choice between two alternative actions $a_i \in A \equiv \{a_1, a_2\}$. For now, the indices i and m each only take on two values, but the notation we develop here will later extend to results involving more players and more actions. Each member m has a continuous utility function u^m that depends on the choice a_i and the state of the world $\varepsilon = (\varepsilon_1, \varepsilon_2) \in \Omega$:

$$u^m(a_i, \varepsilon, x) = \nu_i^m + \varepsilon_i, \ m \in \{h, w\}. \tag{1}$$

This utility function consists of two components, a deterministic term ν_i^m and a state-specific term ε_i, each of which depends on the choice a_i.[10] Both ν_i^m and the distribution of ε_i may also depend on a covariate vector x, containing variables like individual attributes

[9] See footnote 4.

[10] The utility function in equation 1 can be extended to incorporate more complex relationships between ν_i^m and ε_i, as long as u^m monotonically increases with ε_i. We assume additivity between the two parts here for simplicity.

of the spouses (e.g., age, education, income, and health status) or attributes of the household (e.g., whether they rent or own a home). The state of the world ε follows a conditional cumulative distribution function $F(\cdot \mid x)$ with mean normalized to zero. We will usually omit the vector x for notational simplicity.

If an individual m observed ε and then chose a_i to maximize the above utility function, this would be a standard binary choice model—e.g., if F was a standard normal distribution this would yield an ordinary binary probit model. We instead consider a collective household where each spouse m has their own utility function.

2.2 Full symmetric information

Before the realization of ε, a husband and wife collectively design a decision strategy, which can be formulated as $\pi : \Omega \rightarrow \Delta(A)$, a mapping from ε to the set of all probability distributions over A. That is, $\pi(a_i \mid \varepsilon)$ is the probability of the household choosing a_i conditional on ε. After the realization of ε, the husband and wife make a choice based on their ex ante determined strategy. The decision process is illustrated in panel A of Figure 1.

To model the collective household's behavior, we follow Chiappori (1988, 1992) by making the following assumption:

Assumption 1 *The household decision strategy $\pi : \Omega \rightarrow \Delta(A)$ is efficient in the sense that no other feasible choice would have enhanced the utility of both spouses.*

Given the randomness of ε, this assumption posits that household decisions exhibit ex ante efficiency. Whether households actually behave efficiently is an open question—e.g., income hiding and domestic violence are sometimes cited as evidence of inefficiency. Nevertheless, the assumption of efficiency is widely used in both theoretical and empirical models of the household. See, e.g., Browning et al. (1994), Lewbel and Pendakur (2022), and references therein.

2.2.1 Full information equilibrium

We now consider the household's equilibrium behavior. The optimal strategy is to maximize each member m's expected utility while holding the expected utility of the other member m' at a given level, denoted as $u_o^{m'}$:

$$\max_{\pi} \int \sum_{i=1}^{2} \pi\left(a_i \mid \varepsilon\right)\left(\nu_i^m + \varepsilon_i\right) \, \mathrm{d}F(\varepsilon), \tag{2}$$

$$\text{s.t.} \int \sum_{i=1}^{2} \pi\left(a_i \mid \varepsilon\right)\left(\nu_i^{m'} + \varepsilon_i\right) \mathrm{d}F(\varepsilon) \geq u_o^{m'}, \; m \in \{h, w\} \text{ and } m' \neq m. \tag{3}$$

Let λ^m denote the Lagrange multiplier for the constraint in equation 3. Then the problem is equivalent to:

$$\max_{\pi} \sum_{m \in \{h,w\}} \lambda^m \int \sum_{i=1}^{2} \pi\left(a_i \mid \varepsilon\right)\left(\nu_i^m + \varepsilon_i\right) \mathrm{d}F(\varepsilon). \tag{4}$$

Based on equation 4, a Pareto-efficient outcome maximizes a weighted sum of the two individual utilities, with the weight being λ^m for member m. A feature of the formulation in equation 4 is that the Pareto weight λ^m has a natural interpretation in terms of m's intra-household bargaining power (Browning et al., 1994). Since increasing λ^m in equation 4 results in a move along the Pareto set in the direction of higher utility for m and lower for m', the coefficient λ^m reflects m's bargaining power in the sense that a larger λ^m corresponds to more power and better outcomes being enjoyed by m.

Note that our model permits, but does not require, the presence of a vector z of distribution factors, defined as observed household characteristics that affect Pareto weights λ^m but do not affect individual household members' utility functions u^m (or a budget constraint if present). Possible examples of distribution factors include sex ratio on the relevant marriage market, divorce legislation, generosity of single parent benefits, spouses' wealth at marriage, and the targeting of specific benefits to particular members (Browning et al., 1994; Bourguignon et al., 2009).

Let $e \equiv \varepsilon_1 - \varepsilon_2$ and $v^m \equiv \nu_1^m - \nu_2^m$, so, as described in the introduction, the utility from choosing a_1 is $v^m + e$ for member m, with the utility from choosing a_2 being normalized to 0. This is a free normalization for each member, which is taken before applying the above equilibrium calculation. This normalization allows us to reformulate the problem in equation 4 as follows:

$$\max_{\pi} \sum_{m \in \{h,w\}} \lambda^m \int \pi\left(a_1 \mid e\right)\left(v^m + e\right) \mathrm{d}G(e), \tag{5}$$

where $G(\cdot)$ is the cumulative distribution function of e. Here $\pi(a_1 \mid e)$ is the probability that the household chooses a_1 conditional on e; so the probability that the household chooses a_2 is $1 - \pi(a_1 \mid e)$.

For ease of exposition, we normalize the wife's bargaining power to 1 (this is another free normalization) and denote the husband's relative bargaining power as λ^h. Solving problem 5 gives us the following proposition.

Proposition 1 *Under Assumption 1, the household's optimal strategy $\pi^*(a_i \mid e)$ is*

$$\pi^*(a_1 \mid \varepsilon) = \mathbb{1}(e \geq c^*),$$

where $\mathbb{1}(\cdot)$ is an indicator function and

$$c^* = -\frac{\lambda^h v^h + v^w}{\lambda^h + 1}. \tag{6}$$

Proof. See Appendix I. ∎

This result says that the household will choose a_1 when e is above the cutoff c^* defined by equation 6. This cutoff depends on bargaining power and the deterministic utility levels of each household member. If the husband had zero bargaining power, so $\lambda^h = 0$, the household decision would be determined just by the wife's utility, with a cutoff $c^* = -v^w$. In this case the model would reduce to standard binary choice—e.g., a logit model if G has a logistic distribution.

2.2.2 Full information identification

Recall that $\pi(a_1 \mid e)$ is the conditional probability of the household choosing action a_1. Let $p = \int \pi(a_1 \mid e)\, \mathrm{d}G(e)$ denote the unconditional probability that the household chooses a_1. Suppose a researcher has the information that would be used to estimate a logit or probit model. This means that the researcher knows the distribution function G (e.g., logistic if a logit model or standard normal if a probit model), and can estimate the probability p, either by observing the household making repeated choices, or by observing the choices of a homogeneous sample of households.

The household's optimal cutoff c^* in equation 6 could then be identified from $p = 1 - G(c^*)$, assuming that the distribution function G is invertible. However, while c^* is identified, the spouses' bargaining power and utilities—i.e., the parameters λ^h, v^h,

v^w—are not, since we only have one equation 6 and three unknowns.

To illustrate this point, suppose $\{\check{\lambda}^h, \check{v}^h, \check{v}^w\}$ is a solution set. Then $\{(\check{\lambda}^h + 1)(1+\epsilon) - 1,\ \check{v}^h,\ \check{v}^w + (\check{v}^w - \check{v}^h)(1+\epsilon)\}$ forms another solution set, where ϵ is an arbitrarily small positive constant. So a continuum of solutions exist, corresponding to different values of ϵ. Note that this remains true even if we observed some distribution factors z, by replacing $\check{\lambda}^h$ with $\check{\lambda}^h(z)$. This example shows that, unlike the continuous demand model, here we cannot even identify how bargaining power changes in response to a change in a distribution factor.

Although we will not pursue this further, some of the additional assumptions that have been proposed in the previous literature to achieve identification in the continuous demand model could also work to identify the full information discrete model here. For example, Browning et al. (2013) show identification assuming that continuous demand functions are observed for both singles and couples, and that individual's utility functions stay fixed before and after marriage. These additional assumptions would allow us to achieve identification in our model, since the individual's binary choices as singles (such as ordinary logit or probit models) would identify v^h and v^w, and given those parameters along with c^*, the bargaining power λ^h could be recovered from equation 6.

2.3 Asymmetric information

Now we incorporate information asymmetry into the collective model. For simplicity, we focus on the case where the wife first learns the realized value of ε and the husband does not, but later we will also consider the reverse.

Before the state of the world ε is realized, both spouses share a common prior $F(\cdot)$. After the realization of ε, suppose the wife learns the value of ε but her husband remains uninformed. The wife then has the option to either fully disclose this information to her husband, or not, depending on whichever gives her higher utility. In the case of full information disclosure, the analysis is as presented in Section 2.1, since both spouses learn the value of ε upon its realization. The decision process is illustrated in panel B of Figure 1.

2.3.1 Partial information disclosure

Assuming it is the wife, not the husband, who learns the realization of ε, we model partial information disclosure as the wife designing a recommendation strategy $\varpi(a_i \mid \varepsilon)$:

$\Omega \to \Delta(A)$, where $\varpi(a_i \mid \varepsilon)$ is the probability of the wife recommending choice a_i to her husband conditional on ε.[11] Upon the realization of ε, the wife recommends a choice in accordance with $\varpi(a_i \mid \varepsilon)$. The husband, upon receiving his wife's recommendation, updates his belief regarding ε and then determines whether to accept the recommendation. The decision process is illustrated in panel C of Figure 1.

We obtain an equilibrium solution for this model using the Bayesian persuasion framework proposed by Kamenica and Gentzkow (2011). The solution concept is an information sender-preferred subgame perfect equilibrium, since given a prior $F(\cdot)$ and the choice a_i recommended by the wife (information sender), the husband (receiver) forms the posterior $F_{\varpi}(\varepsilon \mid a_i)$ using Bayes's rule and makes a decision that maximizes his utility. In this case, solving the model requires a less stringent version of the ex ante efficiency assumption:

Assumption 1' *The household decision strategy $\pi : \Omega \to \Delta(A)$ is invariant to the state of the world ε.*

This assumption inherently follows Assumption 1, suggesting that the Pareto weights remain invariant across various values of ε (Browning et al., 1994; Browning, 2009; Browning et al., 2014).

To consider equilibrium household behavior in the case of partial information disclosure, we begin by characterizing the husband's problem. Given the wife's recommendation strategy $\varpi(a_i \mid \varepsilon)$, the husband follows her recommendation if and only if

$$\int (\nu_i^h + \varepsilon_i)\, \mathrm{d}F_{\varpi}(\varepsilon \mid a_i) \geq \int (\nu_j^h + \varepsilon_j)\, \mathrm{d}F_{\varpi}(\varepsilon \mid a_i),\ i = 1, 2 \text{ and } j \neq i. \tag{7}$$

That is, when the wife recommends a_i, the husband's expected utility from choosing a_i must exceed his expected utility from choosing a_j. Since the wife has to consider her husband's behavior (equation 7) in making recommendations, this ensures that in equilibrium, the husband will always follow the wife's recommendation. Therefore, the household's final decision will be whatever the wife recommends.

As before, in our world of just two choices, we can simplify notation by letting $e \equiv$

[11] The model allows the sender to choose any form of signal to reveal, not limited to a binary signal from the choice set. Kamenica and Gentzkow (2011) note that focusing on signals from the choice set (in our case, the wife recommends either a_1 or a_2) greatly simplifies the analysis without loss of generality.

$\varepsilon_1 - \varepsilon_2$ and $v^m \equiv \nu_1^m - \nu_2^m$ denote, respectively, the state-specific and deterministic utility associated with choosing a_1 relative to a_2, normalizing the utility of a_2 to zero. We can then replace the vector ε with the scalar e. Since the outcome of the equilibrium is that the household will do whatever the wife recommends, we have

$$\pi(a_1 \mid e) \equiv \varpi(a_1 \mid e).$$

We next establish the following result that simplifies our analysis.

Lemma 1 *The husband's expected utility upon receiving the wife's recommendation is no lower than his utility without such recommendation:*

$$\int \pi(a_1 \mid e)(v^h + e)\ \mathrm{d}G(e) \geq \max\{v^h, 0\}, \tag{8}$$

where $G(\cdot)$ is the cumulative distribution function of e.

Proof. This result is essentially equivalent to equation 7. See Appendix I. ■

2.3.2 Asymmetric information equilibrium

Taking her husband's behavior under partial information disclosure as given, the wife either fully discloses the information e, or chooses a recommendation strategy $\pi(a_1 \mid e)$ before the realization of e. Full information disclosure results in the husband's expected utility being u_o^h defined earlier, and partial information disclosure results in his expected utility being $\max\{v^h, 0\}$. The wife will choose whichever constraint is less restrictive to maximize her expected utility:

$$\max_{\pi} \int \pi(a_1 \mid e)(v^w + e)\ \mathrm{d}G(e), \tag{9}$$

$$\text{s.t.} \int \pi(a_1 \mid e)(v^h + e)\ \mathrm{d}G(e) \geq \min\left\{u_o^h,\ \max\{v^h, 0\}\right\}. \tag{10}$$

That is, the wife will opt for full information disclosure when $u_o^h \leq \max\{0, v^h\}$ and partial information disclosure otherwise. The husband's expected utility will therefore be the lesser of that under full or partial information disclosure.

We again reformulate the problem using a lagrange multiplier that represents bargaining power:

$$\max_{\pi} \ \lambda^h \int \pi(a_1 \mid e)(v^h + e) \, \mathrm{d}G(e) + \int \pi(a_1 \mid e)(v^w + e) \, \mathrm{d}G(e). \tag{11}$$

Solving this problem yields the following proposition.

Proposition 2 *Suppose the wife, and not the husband, learns the value of ε after its realization. The household's optimal strategy $\pi^*(a_i \mid e)$ is then*

$$\pi^* (a_1 \mid e) = \mathbb{1}(e \geq c^*),$$

where $\mathbb{1}(\cdot)$ is the indicator function and c^* depends on the value of u_o^h versus $\max\{v^h, 0\}$ as follows:

i) When $u_o^h \leq \max\{v^h, 0\}$, the wife fully discloses e to the husband under Assumption 1, with c^* given by equation 6;

ii) When $u_o^h > \max\{v^h, 0\}$, the wife recommends a choice and the husband always follows under Assumption 1', with

$$c^* = \begin{cases} k(-v^h) & \text{if } \ v^h > 0 \ \text{and} \ \ k(-v^h) < -v^w, \\ q(-v^h) & \text{if } \ v^h < 0 \ \text{and} \ \ q(-v^h) > -v^w, \\ -v^w & \text{otherwise,} \end{cases} \tag{12}$$

where $k^{-1}(c) \equiv \mathbb{E}\,[e \mid e < c]$ and $q^{-1}(c) \equiv \mathbb{E}\,[e \mid e \geq c]$.[12]

Proof. See Appendix I. ∎

As in the full information case, we have that the household will choose a_1 when the relative utility of doing so is above some cutoff, and hence when $e \geq c^*$. But now the formula that determines c^* is more complicated, as laid out in Proposition 2.

2.3.3 Intra-household bargaining power

Solving the problem in equation 11 also yields the following proposition about intra-household bargaining power.

[12]These definitions ensure that $k(-v^h)$ is defined when $v^h > 0$, and that $q(-v^h)$ is defined when $v^h < 0$. We are assuming that the functions $\mathbb{E}\,[e \mid e < c]$ and $\mathbb{E}\,[e \mid e \geq c]$ are invertible. Note that these conditional expectation functions are themselves fully determined by the cumulative distribution function G.

Proposition 3 *Suppose the wife, and not the husband, learns the value of ε after its realization. Under Assumption 1, the husband's relative bargaining power is*

$$\lambda^h = \min\{\lambda_o^h,\ \lambda^{h*}\},$$

where λ_o^h is the Pareto weight in case i) with full information disclosure and λ^{h*} is the Pareto weight in case ii) with partial information disclosure:

$$\lambda^{h*} = -\frac{v^w + c^*}{v^h + c^*}, \tag{13}$$

where c^* is given by equation 12.

Proof. See Appendix I. ■

Under full information, the husband's bargaining power is some nonnegative value λ_o^h. With asymmetric information, the husband's bargaining power λ^h either still equals λ_o^h (if the wife chooses to reveal ε), or equals λ^{h*} that is determined by c^*, v^h, and v^w using equation 13, and c^* is in turn determined by λ_o^h, v^h, v^w and G, as described in Proposition 2.

To analyze the factors that determine bargaining power, we rewrite equation 13 as:

$$\lambda^{h*} = \frac{v^h - v^w}{v^h + c^*} - 1. \tag{14}$$

This shows that the closer v^h is to v^w (i.e., the more similar are the utilities of the spouses), the lower is the bargaining power of the husband resulting from his information disadvantage. Also, λ^{h*} depends on c^* and c^* depends on $k(\cdot)$ and $q(\cdot)$, which themselves are determined by G. The more disbursed is the distribution G, the smaller will be the absolute values of $k(\cdot)$ or $q(\cdot)$,[13] resulting in lower values of the husband's bargaining power.

In short, the premium in bargaining power for the wife due to her information advantage will be larger when the spouses' preferences are more aligned, or when the state-specific shocks are more dispersed.

[13]To illustrate this point, consider a case where $v^h > 0$ and $k(-v^h) < -v^w$, resulting in $c^* = k(-v^h)$, so $\mathbb{E}[e \mid e < c^*] = -v^h$. A smaller absolute value of $k(\cdot)$ indicates a more dispersed distribution of e. And the smaller the absolute value of $k(\cdot)$ is, for a given value of $-v^h < 0$, the closer is the cutoff c^* (which is negative in sign) to zero, and hence the lower is λ^{h*}.

2.3.4 Identification with asymmetric information

The fact that information asymmetry affects bargaining power, but not the preferences of the individual spouses (or any budget constraint), means that information asymmetry fits the definition of a distribution factor. As discussed earlier, in collective models with continuous demands, observing a distribution factor is not sufficient to identify the level of relative bargaining power from household demand functions (though one can identify how the level of bargaining power changes when distribution factors change). In contrast, a unique feature of our collective model is that the level of relative bargaining power λ^h, as well as spouses' utilities v^h and v^w, may be identified from household choices given the presence of information asymmetry as a distribution factor.

As before, suppose that the wife learns the value of e after its realization but the husband does not, and suppose $v^h > 0$ (results when the husband learns e first and/or $v^h < 0$ are analogous).[14] In this case, when there is an incentive for the wife to partially disclose information, the optimal cutoff is:

$$c^* = \begin{cases} k(-v^h) & \text{if } \; k(-v^h) < -v^w, \\ -v^w & \text{otherwise.} \end{cases} \tag{15}$$

As in the full information case, we assume that a researcher has the information that would be used to estimate a logit or probit model. This means that the researcher knows G, the distribution function of e (e.g., logistic if a logit model or standard normal if a probit model), and can estimate p, the unconditional probability that a household chooses action a_1 (either by observing the household making repeated choices, or by observing the choices of a homogeneous sample of households). As in the full symmetric information case, the household chooses a_i if e exceeds a cutoff c^*, so c^* is identified by $p = 1 - G(c^*)$, assuming that the distribution function G is invertible. However, unlike the symmetric case, c^* is now given by equation 15. Note that the function k is determined by G, so the function k is known by the researcher.

Similarly, if it is the husband instead of the wife who observes e, and if he only

[14] We can also allow for the value of v^h versus 0 to depend on covariates x. An example will be in our case study.

partially discloses information, and $v^w > 0$ then

$$c^* = \begin{cases} k(-v^w) & \text{if } \ k(-v^w) < -v^h, \\ -v^h & \text{otherwise.} \end{cases} \tag{16}$$

Equations 15 and 16 provide two ways of identifying v^h. By identifying c^* for households in the first scenario of equation 15 we can identify v^h since in that case $c^* = k(-v^h)$. Alternatively, we can identify v^h by identifying c^* for households in the second scenario of equation 16, since then $c^* = -v^h$. Similarly, we can identify v^w either from the second scenario of equation 15 or from the first scenario of equation 16. Finally, given v^h, v^w, and c^*, we can identify the level of bargaining power λ^h by Proposition 3.

In summary, spouses' bargaining power and preferences can be identified using our model if, in at least some households, one spouse has an information advantage and has an incentive to partially disclose that information. If w always has the information advantage, identification requires that some households have $v^w < -k(-v^h)$ and some have $v^w > -k(-v^h)$. But if either spouse can sometimes have an information advantage (as will be the case in our application), then we only need either $v^m < -k(-v^{m'})$ or $v^m > -k(-v^{m'})$ for $m \in \{h, w\}$ to hold.

Note that this identification requires observing which scenario a household is in. One way this could be accomplished is via covariates. For example, in our application, whichever spouse migrates away is assumed to have an information disadvantage regarding their child's abilities, relative to the spouse that lives at home with the child. Further, suppose in this case that some covariate x^h (e.g., the husband's age, or the price to the husband of choosing a_1) is known to affect v^h and not v^w. Then the households where the wife is exploiting an information advantage and are observed to have c^* varying with x^h must be households in the first scenario of equation 15, and those where c^* does not vary with x^h are in the second scenario of equation 15.

Remark To achieve identification, we assume that the researcher possesses the necessary information to estimate a logit or probit type model—i.e., knowledge of the distribution function G. A potential concern is bias in identification and estimation if the researcher makes incorrect assumptions regarding the distribution G. We address this concern in the following section, showing that the bias resulting from an incorrect distribution assumption is generally small, and if it becomes significant, a statistical test can assist in selecting the correct specification of G.

3 A case study of child development in households with a migrant parent

In this section, we apply our collective model with information asymmetry to analyze investment decisions within migrant households. We show that our model can elucidate certain puzzling empirical patterns regarding child development in households with one migrant parent.

3.1 Stylized facts

The out-migration of parents has become a common childhood experience worldwide. In 2020, the number of international migrants reached close to 281 million. Migration within a country occurs even more frequently, particularly in China, where approximately 170 million people worked outside their home areas for more than half a year in 2020.[15] Due to restrictions on migrant access to local health and education systems, children are often left behind when their parent(s) migrate for work. In 2020, there were 67 million left-behind children in China, constituting more than 22 percent of all children. The well-being of these children is a significant concern.

Puzzling patterns from fathers' versus mothers' migration While existing research highlights the adverse effects of parental migration on child development (e.g., Zhang et al., 2014), it suggests that children typically fare better when mothers stay at home compared to when fathers stay. For example, Chen et al. (2014) find that in rural China, an overall insignificant effect of parental migration on educational performance masks different effects for fathers and mothers: compared to those with both parents at home, children whose mothers migrate have worse performance, whereas they show improved performance when only the father migrates. Yue et al. (2020) highlight the adverse effect of the mother's migration on younger children's cognitive development. Similar patterns are observed in other countries. Antman (2012) reveals improved school-

[15]Data are from the World Migration Report (https://digitallibrary.un.org/record/3951157?ln=en), and the National Bureau of Statistics of China (http://www.stats.gov.cn/sj/zxfb/202302/t20230203_1901074.html).

ing outcomes for Mexican girls with their father's US migration. Lu (2014) shows that the negative association between parental migration and child development in Mexico is mostly evident in families where the mother migrates, and in Indonesia, children face a significant disadvantage when the mother migrates.

Channels based on existing research for how parental migration affects child development include (i) a detrimental impact from the absence of parents and (ii) a positive impact from increased family income that enables greater investment in children. For the former channel, see Lyle (2006), McKenzie and Rapoport (2011), and Zhang et al. (2014). For the latter channel, see Gibson et al. (2011), Antman (2013), and Gibson and McKenzie (2014). These two channels, however, provide limited insight into the patterns when separately considering fathers' versus mothers' migration. In particular, we find that children faring better when fathers versus mothers migrate holds in China even after conditioning on income—i.e., this pattern is not just due to migrant fathers earning more than migrant mothers.

Information asymmetry and an investment channel We propose a novel channel based on our collective model with information asymmetry that can account for these patterns. In migrant households, there exists asymmetric information regarding children's talents and development potential: the parent staying at home with the child likely possesses better information, while the migrating parent is less informed. Assuming that on average wives are more inclined than husbands to sacrifice consumption for child investment, it follows that if she stays at home and has the information advantage, she will ceteris paribus invest more in the child, leading to better child development outcomes.

To investigate this investment level channel, we empirically examine the relationship between parental migration and child educational expenditure using cross-sectional data from the China Family Panel Studies survey. This is a nationally representative household survey administered by Peking University's Institute of Social Science Survey, covering 25 out of 34 provinces (Institute of Social Science Survey, 2015). We utilize its baseline wave from 2010. Results are presented in Table 1. The sample comprises rural families with at least one child aged 6–15. Due to China's compulsory schooling law, mandating all children to enroll in school at the age of six to complete nine years of compulsory education, there is limited variation in expenditure for formal schooling.

We thus use yearly extracurricular educational expenditure per child as the dependent variable.[16] Families in which both parents migrate are excluded (less than two percent of the sample). Regressions incorporate controls for whether only the father migrates and whether only the mother does. Families with both parents staying at home serve as the reference group.

We find that father migration is positively correlated with child extracurricular educational expenditure, while mother migration is negatively correlated, though not statistically significant. These patterns hold for one-child families (columns 1 and 2) and for first-born children (columns 3 and 4), and remain even when household total income is controlled for.[17] In terms of magnitude, father migration is associated with a roughly 50 percent increase in this expenditure. Given its mean of 251 yuan in the sample, this translates to a 125.5 yuan increase. Average household income is 13,307 yuan, so this 125.5 yuan increase represents about one percent of household income. For families with a migrant parent, the average migration remittance is 8,476 yuan, and 125.5 yuan constitutes about 1.5 percent of migration remittance.

3.2 Applying the model

Suppose parents (a husband and wife) face a decision between making a high-level child investment a_1 or a lower-level investment a_2. The prices of these investments as a proportion of total income are φ_1 and φ_2, respectively, with $\varphi_1 > \varphi_2$. Here φ_1 and φ_2 are examples of covariates x that can affect utility levels.

Parental utility function Parents derive utility from consumption and from child quality, with a utility function expressed by equation 1. Opting for a high-priced investment a_1 results in lower parental consumption, but is expected to yield higher child quality. The husband and wife exhibit distinct levels of (dis)utility from forgoing consumption due to investment choice a_i, denoted by ν_i^h and ν_i^w, while sharing the same level of utility from child quality, denoted by ε_i.

[16] This includes private tutoring fees, book expenses, and accommodation fees—but excludes formal education fees such as tuition.

[17] Column 1 includes baseline controls: the age and gender of the child, and the age and schooling years of both parents. Column 2 further includes household income. Column 3 includes baseline controls along with the number of children in the family. Column 4 further includes household income. All columns control for county fixed effects.

For simplicity, we adopt the following functional form for ν_i^m:

$$\nu_i^m = (\gamma^m)^{-1} \log(1 - \varphi_i), \ m \in \{h, w\},$$

where γ^m reflects how parent m values the forgone consumption due to the cost of child investment, with a higher value indicating a greater willingness to invest resources in the child. Normalizing γ^w to 1, we assume that $\gamma^h < 1$, implying that the husband is less inclined to sacrifice current consumption for child investment. One reason could be evolutionary: men have a longer reproductive horizon, and so may wish to spend less on a current child and more on potential future children (and partners). See Biblarz and Raftery (1999) and Trivers (2017).

The random component of utility ε_i represents child development resulting from parents' choice of a_i, with its mean normalized to zero. In other words, the expected value of the enhancement in child development associated with opting for the higher investment over choosing the lower one is set to zero. Let $e \equiv \varepsilon_1 - \varepsilon_2$. A more gifted child would exhibit a higher value of e compared to a less gifted child. As before, the utility of choosing a_2 is normalized to zero.

Equilibrium under partial information disclosure We first consider the case where the wife stays home with the child and learns the value of ε (and hence e), while the husband migrates and so is uninformed. In the case of partial information disclosure, the wife recommends an investment choice and the husband consistently agrees under Assumption 1'. It then follows from Proposition 2 that the household will choose action a_1 if e is above the following cutoff:

$$c^* = \max \left\{ q \left(-(\gamma^h)^{-1} \log \frac{1 - \varphi_2}{1 - \varphi_1} \right), \ -\log \frac{1 - \varphi_2}{1 - \varphi_1} \right\}, \tag{17}$$

where $q^{-1}(c) \equiv \mathbb{E}[e \mid e \geq c]$. By Proposition 3 the husband's relative bargaining power is then:

$$\lambda^{h*} = -\frac{\log \frac{1-\varphi_2}{1-\varphi_1} + c^*}{(\gamma^h)^{-1} \log \frac{1-\varphi_2}{1-\varphi_1} + c^*}, \tag{18}$$

where c^* is given by equation 17.

It follows directly from these equations that the greater is the difference between φ_1

and φ_2, the higher is the cutoff c^*, meaning a reduced likelihood of choosing the higher-priced investment a_1, resulting in higher bargaining power for the husband. Below we provide more implications of these propositions.

3.3 Model implications for bargaining power, investment, and child development

Figure 2 illustrates the relative bargaining power of the husband.[18] The dark solid line depicts the case under partial information disclosure, with $\gamma^h < \gamma^w$, where the wife learns e while the husband migrates and so is uninformed. The husband's relative bargaining power monotonically decreases with the variance of e, consistent with our derivation that the wife's premium in bargaining power due to her information advantage is greater when the distribution of state-specific shocks (in this scenario, the child's talents) is more dispersed. The light solid line illustrates the case with an even smaller γ^h, showing that for a given variance of e, the husband possesses more bargaining power as γ^h decreases relative to γ^w. This demonstrates our finding that the wife's premium in bargaining power is smaller when spouses' preferences are less aligned. When spouses' preferences are perfectly aligned, so $\gamma^h = \gamma^w$, the husband has zero bargaining power (but also does not require any power, since in this case maximizing her utility is equivalent to maximizing his). This case is depicted in Figure 2 by the dash-dotted line. Finally, the dashed line depicts the case with symmetric information (as could happen, e.g., when neither parent migrates). In this case the husband has bargaining power that does not depend on the state of e.

Figure 3 illustrates the level of investment in the child,[19] again assuming $\gamma^h < \gamma^w$. The solid line depicts the case as before where the wife learns the value of e while the husband is uninformed. The investment level in this case is higher than the level under symmetric information, which is depicted by the dashed line. This pattern reflects the bargaining power premium of the wife due to her information advantage, resulting in a greater likelihood of choosing the higher-priced a_1. The dotted line depicts the opposite case, where instead the husband learns the value of e while the wife migrates and is thus uninformed, still assuming $\gamma^h < \gamma^w$. In this scenario, the investment level is lower

[18]Parameter values for the illustrations in Figures 2 to 4 are: $\varphi_1 = 0.1$, $\varphi_2 = 0.02$, $\gamma^w = 0.08$, $\gamma^h = 0.04$ when $\gamma^h < \gamma^w$, and $\gamma^h = 0.035$ when $\gamma^h \ll \gamma^w$. In addition, e follows a normal distribution with mean 0 and variance in the range $[1, 5]$.

[19]The level of investment is calculated as $\int [\varphi_1 \, \pi^*(a_1 \mid e) + \varphi_2 \, \pi^*(a_2 \mid e)] \; \mathrm{d}G(e)$.

than the level under symmetric information. In all three cases, the level of investment monotonically increases with the variance of e, indicating a higher investment as the child talent shocks become more dispersed.

Figure 4 illustrates the level of child development.[20] In line with the investment patterns observed in Figure 3, the level of child development is higher when the husband migrates while the wife stays at home. Conversely, when the wife migrates and the husband stays with the child, the level of child development is lower.

Overall, our model's implications align with the empirically observed patterns of child development outcomes arising from fathers' versus mothers' migration (and the differing patterns of educational investment we find in Table 1), discussed in Section 3.1. One could imagine alternative explanations that our model does not address: e.g., wives may make better use of existing resources in nurturing children's development, such as spending more time with children. However, while a mother's time spent with the child can be a resource, it is hard to see why a father living elsewhere would experience (dis)utility from that. The results in Table 1 reflect correlation and not necessarily causality, so another possible explanation for the differing patterns of educational investment could be selection—i.e., unobserved child talents might influence the likelihood of mothers' and fathers' migration in a different manner. However, justifying why child talents would differentially affect these migration decisions is not straightforward.

3.4 Bias with (incorrect) distribution assumptions

Our model assumes that the researcher knows the distribution function G. Here we perform a numerical illustration to assess the bias in recovered parameters $(\gamma^w)^{-1}$, $(\gamma^h)^{-1}$, and λ_o^h/λ_o^w, as well as in outcomes λ^h/λ^w, investment, and child development, if the researcher assumes the wrong distribution function G. Results are in Table 2. Specifically, we generate the data with normal, logistic, and extreme value type I errors. In each case we assess the bias and root mean square error (RMSE) when the assumed distribution is normal, logistic, or extreme. We also consider scenarios with low and high variance in panels A and B.[21]

We also show it is possible to test alternative assumptions regarding G. In particular, we apply the Vuong (1989) test, where the null hypothesis is that competing models are

[20] The level of child development is calculated as $\int [\varepsilon_1\ \pi^*(a_1 \mid e) + \varepsilon_2\ \pi^*(a_2 \mid e)]\ \mathrm{d}F(\varepsilon)$.

[21] That is, the variance of e is set to either 2 or 4.

equally close to the true data generating process while the alternative is that one model is closer. The power of this test is reported in the final row of each panel of Table 2, showing that the null is mostly rejected when it should be.

In summary, we find that the bias resulting from an incorrect distribution assumption made by the researcher is generally small. And in cases where the difference in choice of G is significant, the Vuong test can assist the researcher in making informed assumptions about the distribution.

4 Extensions

In this section, we extend our model in Section 2 to situations with multiple choices and then to multiple players.

4.1 Household collective decisions with multiple choices

Suppose the husband and wife now face a choice from a finite set of I alternative actions: $a_i \in A \equiv \{a_1, a_2, ..., a_I\}$. Their utility function is given by equation 1, which depends on the choice a_i and the state of the world ε. The household's collective decision $\pi(a_i \mid \varepsilon)$ can be obtained by solving a straightforward extension of the problem in equation 4:

$$\max_{\pi} \sum_{m\in\{h,w\}} \lambda^m \int \sum_{i=1}^{I} \pi(a_i \mid \varepsilon)(\nu_i^m + \varepsilon_i)\, \mathrm{d}F(\varepsilon), \tag{19}$$

where λ^m represents m's bargaining power.

For any two different choices a_i and a_j in A, let $e_{ij} \equiv \varepsilon_i - \varepsilon_j$ and $v_{ij}^m \equiv \nu_i^m - \nu_j^m$ denote the state-specific utility and m's deterministic utility associated with choosing the former relative to the latter. The decision $\pi(a_i \mid \varepsilon)$ satisfies:

$$\max_{\pi} \sum_{m\in\{h,w\}} \lambda^m \int \pi(a_i \mid e_{ij})(v_{ij}^m + e_{ij})\, \mathrm{d}G_{ij}(e_{ij}),\ \forall\, j \neq i, \tag{20}$$

where $G_{ij}(\cdot)$ is the cumulative distribution function of e_{ij}. Normalizing the wife's bargaining power to 1 and denoting the husband's as λ^h, we obtain a result parallel to Proposition 1.

Proposition 4 *Under Assumption 1, the household's optimal strategy* $\pi^*(a_i \mid \varepsilon)$ *is*

$$\pi^*\left(a_i \mid \varepsilon\right) = \mathbb{1}\left(\bigcap_{j\neq i}^{I}\{e_{ij} \geq c_{ij}^*\}\right),$$

where $\mathbb{1}(\cdot)$ *is an indicator function and*

$$c_{ij}^* = -\frac{\lambda^h v_{ij}^h + v_{ij}^w}{\lambda^h + 1}. \tag{21}$$

Proof. See Appendix II for the proof of this proposition and all subsequent ones. These proofs represent straightforward extensions of the ones provided in Section 2. ■

We then suppose that the wife learns the value of ε upon its realization but the husband remains uninformed. In the case of partial information disclosure where the wife recommends a choice a_i, the husband upon receiving the recommendation updates his belief regarding ε and then decides whether to accept. The wife's recommendation strategy serves as the equilibrium decision, as she formulates her recommendation to ensure that husband follows it.

Taking the husband's behavior in this case as given, the wife either fully discloses the information to her husband or chooses a recommendation strategy $\pi(a_i \mid \varepsilon)$, depending on which option maximizes her expected utility:

$$\max_{\pi} \int \pi(a_i \mid e_{ij})(v_{ij}^w + e_{ij})\ \mathrm{d}G_{ij}(e_{ij}), \tag{22}$$

$$\text{s.t.} \int \pi(a_i \mid e_{ij})(v_{ij}^h + e_{ij})\ \mathrm{d}G_{ij}(e_{ij}) \geq \min\left\{u_{o,ij}^h,\ \max\{v_{ij}^h, 0\}\right\},\ \forall\ j \neq i, \tag{23}$$

where $u_{o,ij}^h$ is the husband's expected utility under full information disclosure and $\max\{v_{ij}^h, 0\}$ is his utility under partial information disclosure. These equations are parallel to equations 9 and 10. Solving the problem yields a result parallel to Proposition 2.

Proposition 5 *Suppose the wife, and not the husband, learns the value of* ε *after its realization. The household's optimal strategy* $\pi^*(a_i \mid \varepsilon)$ *is*

$$\pi^*\left(a_i \mid \varepsilon\right) = \mathbb{1}\left(\bigcap_{j\neq i}^{I}\{e_{ij} \geq c_{ij}^*\}\right),$$

where $\mathbb{1}(\cdot)$ *is an indicator function and* c^*_{ij} *depends on the value of* $u^h_{o,ij}$ *versus* $\max\{v^h_{ij}, 0\}$*:*

i) When $u^h_{o,ij} \leq \max\{v^h_{ij}, 0\}$*, the wife fully discloses* e_{ij} *to her husband under Assumption 1, with* c^*_{ij} *given by equation 21;*

ii) When $u^h_{o,ij} > \max\{v^h_{ij}, 0\}$*, the wife recommends a choice from* $\{a_i, a_j\}$ *and the husband always follows under Assumption 1', with*

$$c^*_{ij} = \begin{cases} k_{ij}(-v^h_{ij}) & \textit{if } \ v^h_{ij} > 0 \ \textit{and} \ \ k_{ij}(-v^h_{ij}) < -v^w_{ij}, \\ q_{ij}(-v^h_{ij}) & \textit{if } \ v^h_{ij} < 0 \ \textit{and} \ \ q_{ij}(-v^h_{ij}) > -v^w_{ij}, \\ -v^w_{ij} & \textit{otherwise}, \end{cases} \tag{24}$$

where $k^{-1}_{ij}(c) \equiv \mathbb{E}\left[e_{ij} \mid e_{ij} < c\right]$ *and* $q^{-1}_{ij}(c) \equiv \mathbb{E}\left[e_{ij} \mid e_{ij} \geq c\right]$*.*

Note that the Lagrange multiplier for the constraint in equation 23 may depend on specific choices a_i and a_j. To solve for intra-household bargaining power that is invariant to the choices, we impose an additional assumption about spouses' utility functions.

Assumption 2 *For any two choices* a_i *and* a_j *in* A*:*

1. Spouses' preferences differ by a constant scale, such that $v^w_{ij} = bv^h_{ij}$*;*

2. The distribution of normalized random utility, expressed as $\tilde{e}_{ij} \equiv e_{ij}/v^h_{ij}$*, is identical.*

We then obtain the following proposition about bargaining power.

Proposition 6 *Suppose the wife, and not the husband, learns the value of* ε *after its realization. Under Assumptions 1 and 2, the husband's relative bargaining power is*

$$\lambda^h = \min\{\lambda^h_o, \ \lambda^{h*}\},$$

where λ^h_o *is the Pareto weight in case i) with full information disclosure; and* λ^{h*} *is the Pareto weight in case ii) with partial information disclosure:*

$$\lambda^{h*} = \max\left\{0, \ -\frac{b - \tilde{q}(1)}{1 - \tilde{q}(1)}\right\}, \tag{25}$$

where $\tilde{q}^{-1}(c) \equiv \mathbb{E}\left[\tilde{e}_{ij} \mid \tilde{e}_{ij} \geq c\right]$*.*

We arrive at similar conclusions about bargaining power as before. The wife's premium in bargaining power due to her information advantage would be larger if spouses' preferences are more aligned (indicated by a smaller difference between b and 1), or if the state-specific shocks are more dispersed (indicated by a smaller value of $\tilde{q}$).

Identification The identification of the model with multiple choices can be decomposed into C_I^2 problems, where I is the number of choices. Each problem is to identify the values of relative bargaining power λ^h and spouses' utilities v_{ij}^h and v_{ij}^w, for a pair of choices $\{a_i, a_j\}$. This can be achieved as discussed in Section 2.3.4. In particular, each problem requires that in at least some households, one spouse has an information advantage and has an incentive to partially disclose the information. Moreover, the researcher needs to be able to classify households by their choices and by who has the information advantage, either through direct observation or through covariates as before. It is worth noting that, as a result of Assumption 2, all problems yield the same λ^h.[22] By not making this assumption, our model enables the identification of intra-household bargaining power, which is specific to each choice pair. This approach could yield more generalized findings in the collective model literature.

4.2 Collective decisions with multiple choices and multiple players

Consider a game that consists of a finite number of players $m \in \Phi$, who face multiple choices from $A \equiv \{a_1, a_2, ..., a_I\}$. The collective decision $\pi(a_i \mid \varepsilon)$ satisfies:

$$\max_{\pi} \sum_{m \in \Phi} \lambda^m \int \pi(a_i \mid e_{ij}) (v_{ij}^m + e_{ij}) \, \mathrm{d}G_{ij}(e_{ij}), \ \forall j \neq i, \tag{26}$$

as parallel to equation 20. Solving this problem gives us the following result.

Proposition 7 *Under Assumption 1, the household's optimal strategy $\pi^*(a_i \mid \varepsilon)$ is*

$$\pi^*(a_i \mid \varepsilon) = \mathbb{1}\left(\bigcap_{j \neq i}^{I} \{e_{ij} \geq c_{ij}^*\}\right),$$

[22]Assumption 2 is to ensure that bargaining power is invariant to the choices. However, on its own, it does not lead to identification: e.g., if $\{\check{\lambda}^h(z), \check{b}, \check{v}^h\}$ is a solution set, then $\{(\check{\lambda}^h + 1)(1 + \epsilon) - 1, \check{b} + (\check{b} - 1)(1 + \epsilon), \check{v}^h\}$ forms another set, where ϵ is an arbitrarily small positive constant.

where $\mathbb{1}(\cdot)$ *is an indicator function and*

$$c_{ij}^{*} = -\frac{\sum_{m\in\Phi} \lambda^{m} v_{ij}^{m}}{\sum_{m\in\Phi} \lambda^{m}}. \tag{27}$$

Suppose some players of the game (call them senders) learn the value of ε after it is realized while other players (call them receivers) remain uninformed. Let S denote the set of senders and $R \equiv \Phi \setminus S$ denote the set of receivers. In the case of partial information disclosure, we represent senders' recommendation strategy as $\pi(a_i \mid \varepsilon)$, which also serves as the final decision. This is because, in equilibrium, receivers consistently follow the recommendation from senders, who take into account receivers' behavior when formulating their recommendations.[23]

Similar to the result in Lemma 1, receivers' expected utility, weighted by the Pareto weights, upon receiving the senders' recommendation is no lower than their weighted utility without such recommendation:

$$\sum_{m\in R} \lambda_{o}^{m} \int \pi(a_i \mid e_{ij})(v_{ij}^{m} + e_{ij}) \, \mathrm{d}G_{ij}(e_{ij}) \geq \max\{v_{ij}^{R}, 0\}, \tag{28}$$

where λ_{o}^{m} is the Pareto weight for $m \in R$ under symmetric information, and $v_{ij}^{R} = \sum_{m\in R} \lambda_{o}^{m} v_{ij}^{m}$.

Taking receivers' behavior under partial information disclosure as given, senders either fully disclose the information so that receivers' expected utility is $u_{o,ij}^{R}$, or senders choose a recommendation strategy such that receivers' expected utility is $\max\{v_{ij}^{R}, 0\}$. Senders will opt for the less restrictive constraint to maximize their weighted utility:

$$\max_{\pi} \sum_{m\in S} \lambda_{o}^{m} \int \pi(a_i \mid e_{ij})(v_{ij}^{m} + e_{ij}) \, \mathrm{d}G_{ij}(e_{ij}), \tag{29}$$

$$\text{s.t.} \sum_{m\in R} \lambda_{o}^{m} \int \pi(a_i \mid e_{ij})(v_{ij}^{m} + e_{ij}) \, \mathrm{d}G_{ij}(e_{ij}) \geq \min\left\{u_{o,ij}^{R}, \ \max\{v_{ij}^{R}, 0\}\right\}, \ \forall\, j \neq i. \tag{30}$$

Senders will choose full information disclosure when $u_{o,ij}^{R} \leq \max\{v_{ij}^{R}, 0\}$ and partial information disclosure otherwise. Receivers' expected utility is the lesser of that under full

[23] The assumption of ex-ante efficiency implies that senders collaboratively design a recommendation strategy, and receivers collectively decide whether to accept. Note that information asymmetry does not affect the relative bargaining power among senders or among receivers.

or partial information disclosure. Solving this problem yields the following proposition.

Proposition 8 *Suppose some players are senders who learn the value of ε after its realization but others are receivers who remain uninformed. The optimal strategy $\pi^*(a_i \mid \varepsilon)$ is*

$$\pi^* (a_i \mid \varepsilon) = \mathbb{1} \left(\bigcap_{j \neq i}^{I} \{ e_{ij} \geq c_{ij}^* \} \right),$$

where $\mathbb{1}(\cdot)$ is an indicator function and c_{ij}^* depends on the value of $u_{o,ij}^R$ versus $\max\{v_{ij}^R, 0\}$:

i) When $u_{o,ij}^R \leq \max\{v_{ij}^R, 0\}$, senders fully disclose e_{ij} to receivers under Assumption 1, with c_{ij}^* given by equation 27;

ii) When $u_{o,ij}^R > \max\{v_{ij}^R, 0\}$, senders recommend a choice from $\{a_i, a_j\}$ and receivers always follow under Assumption 1', with

$$c_{ij}^* = \begin{cases} k_{ij}\left(-\bar{v}_{ij}^R\right) & \text{if } v_{ij}^R > 0 \text{ and } k_{ij}\left(-\bar{v}_{ij}^R\right) < -\bar{v}_{ij}^S, \\ q_{ij}\left(-\bar{v}_{ij}^R\right) & \text{if } v_{ij}^R < 0 \text{ and } q_{ij}\left(-\bar{v}_{ij}^R\right) > -\bar{v}_{ij}^S, \\ -\bar{v}_{ij}^S & \text{otherwise,} \end{cases} \tag{31}$$

where $v_{ij}^R = \sum_{m \in R} \lambda_o^m v_{ij}^m$, $\bar{v}_{ij}^R = v_{ij}^R / \sum_{m \in R} \lambda_o^m$, and $\bar{v}_{ij}^S$ is similarly defined; $k_{ij}^{-1}(c) \equiv \mathbb{E}\left[e_{ij} \mid e_{ij} < c\right]$ and $q_{ij}^{-1}(c) \equiv \mathbb{E}\left[e_{ij} \mid e_{ij} \geq c\right]$.

The following assumption is then introduced to achieve bargaining power that remains invariant to the choices.

Assumption 2' *For any two choices a_i and a_j in A:*

1. Players' preferences differ by a constant scale, such that $v_{ij}^m = b^m v_{ij}^{y_0}$, $\forall\ m \in \Phi$, where y_0 is a random fixed player;

2. The distribution of normalized random utility, expressed as $\tilde{e}_{ij} \equiv e_{ij} / v_{ij}^{y_0}$, is identical.

Normalizing senders' bargaining power to λ_os that arise under symmetric information, we obtain the following result.

Proposition 9 *Suppose some players are senders who learn the value of ε after its realization but others are receivers who remain uninformed. Under Assumptions 1 and 2',* *receiver m's relative bargaining power is*

$$\lambda^m = \min\{\lambda_o^m,\ \lambda^{m*}\},$$

where λ_o^m *is the Pareto weight in case i) with full information disclosure; and* λ^{m*} *is the Pareto weight in case ii) with partial information disclosure:*

$$\lambda^{m*} = \max\left\{0,\ -\lambda_o^m\ \frac{b^S - \sum_{m\in S}\lambda_o^m\ \tilde{q}(\bar{b}^R)}{b^R - \sum_{m\in R}\lambda_o^m\ \tilde{q}(\bar{b}^R)}\right\}, \tag{32}$$

where $b^R = \sum_{m\in R}\lambda_o^m b^m$, $\bar{b}^R = b^R/\sum_{m\in R}\lambda_o^m$, *and* b^S *is similarly defined;* $\tilde{q}^{-1}(c) \equiv \mathbb{E}\left[\tilde{e}_{ij} \mid \tilde{e}_{ij} \geq c\right]$.

We observe that senders' premium in bargaining power due to information advantage would be larger if the preferences of senders and receivers are more aligned (a smaller difference between b^S and b^R), or if the state-specific shocks are more dispersed (a smaller value of $\tilde{q}$).

Identification As discussed in Section 4.1, the identification of the model with I choices can be broken down into C_I^2 problems. Each problem focuses on identifying the values of game players' bargaining power λ^m and preferences v_{ij}^m for a pair of choices $\{a_i, a_j\}$. All problems yield the same values of bargaining power due to Assumption 2'.

We now discuss the conditions needed for identifying one problem, omitting subscripts i and j for notational simplicity. Consider a game with N players. There are N parameters for their preferences and $N-1$ parameters for bargaining power (with player y_0's bargaining power normalized to 1). Thus, $2N-1$ moment conditions are required for identification. We note that there are $2^N - 2$ ways to partition the set of players into senders and receivers, since each of the N players can be either a sender or a receiver, and since information asymmetry excludes the case with no senders or no receivers. For each way of partitioning, the cutoff under partial information disclosure, focusing on the case when $v^R > 0$ as before, is:

$$c^* = \begin{cases} k(-\bar{v}^R) & \text{if } \ k(-\bar{v}^R) < -\bar{v}^S, \\ -\bar{v}^S & \text{otherwise.} \end{cases} \tag{33}$$

This yields two moment conditions, giving us a total of $2 \times (2^N - 2)$ moments. Since one of the moments in equation 33 with a set $\hat{S}$ being the set of senders is equivalent to the other moment when $\hat{S}$ is the set of receivers, the number of independent moment conditions is $2^N - 2$ under partial information disclosure. In addition, equation 27 provides another moment condition under full information disclosure (or symmetric information). In total, there are $2^N - 1$ independent moment conditions, and any subset of $2N - 1$ of them can achieve identification.

5 Conclusion

We incorporate information asymmetry into the collective model, by introducing a random component of utility. This allows one decision-maker to gain information on the random state while the other remains uninformed. By formulating the decision process under partial information disclosure using the Bayesian persuasion framework, we can solve for decision-makers' relative bargaining power and utilities. Notably, our model features identifiable bargaining power, the level of which is endogenous to the decision-maker's information advantage. The analysis reveals a bargaining power premium for the one with information advantage, especially when preferences align more or the state-specific shocks disperse more. We apply our model to analyze investment decisions and child development in households with a migrant parent. Simulation results support the model implications, revealing higher expected investment and child development levels when the wife but not the husband is informed about their child's inherent abilities.

Our model extends to multiple choices and multiple players, with some informed while others remain uninformed. Therefore, our model yields valuable insights into collective behavior across diverse real-world settings where one group of people seek to influence another by offering advice and shaping their beliefs. Possible examples include scenarios like teachers versus students, government agencies versus citizens, managers versus shareholders, marketing professionals versus consumers, healthcare providers versus patients, lobbyists versus politicians, among others.

References

Antman, F. M. (2012). Gender, educational attainment, and the impact of parental migration on children left behind. *Journal of Population Economics 25*, 1187–1214.

Antman, F. M. (2013). The impact of migration on family left behind. *International Handbook on the Economics of Migration*, 293–308.

Apedo-Amah, M., H. Djebbari, and R. Ziparo (2020). Gender, information and the efficiency of household production decisions: An experiment in rural Togo. *Working Paper*.

Ashraf, N. (2009). Spousal control and intra-household decision making: An experimental study in the Philippines. *American Economic Review 99*(4), 1245–1277.

Ashraf, N., E. Field, A. Voena, and R. Ziparo (2022). Gendered spheres of learning and household decision making over fertility. *Working Paper*.

Bargain, O. and O. Donni (2012). Expenditure on children: A rothbarth-type method consistent with scale economies and parents' bargaining. *European Economic Review 56*(4), 792–813.

Becker, G. S. (1981). *A Treatise on the Family.* Cambridge: Harvard University Press.

Biblarz, T. J. and A. E. Raftery (1999). Family structure, educational attainment, and socioeconomic success: Rethinking the 'pathology of matriarchy'. *American Journal of Sociology 105*(2), 321–365.

Blundell, R., P.-A. Chiappori, and C. Meghir (2005). Collective labor supply with children. *Journal of Political Economy 113*(6), 1277–1306.

Bourguignon, F., M. Browning, and P.-A. Chiappori (2009). Efficient intra-household allocations and distribution factors: Implications and identification. *Review of Economic Studies 76*(2), 503–528.

Browning, M. (2009). Love, betrayal, and commitment. *University of Oxford*.

Browning, M., F. Bourguignon, P.-A. Chiappori, and V. Lechene (1994). Income and outcomes: A structural model of intrahousehold allocation. *Journal of Political Economy 102*(6), 1067–1096.

Browning, M., P.-A. Chiappori, and A. Lewbel (2013). Estimating consumption economies of scale, adult equivalence scales, and household bargaining power. *Review of Economic Studies 80*(4), 1267–1303.

Browning, M., P.-A. Chiappori, and Y. Weiss (2014). *Economics of the Family*. Cambridge University Press.

Castilla, C. (2019). What's yours is mine, and what's mine is mine: Field experiment on income concealing between spouses in India. *Journal of Development Economics 137*, 125–140.

Castilla, C. and T. Walker (2013). Is ignorance bliss? the effect of asymmetric information between spouses on intra-household allocations. *American Economic Review 103*(3), 263–268.

Chau, T. W., L. Hongbin, P. W. Liu, and J. Zhang (2007). Testing the collective model of household labor supply: Evidence from china. *China Economic Review 18*(4), 389–402.

Chen, X., Q. Huang, S. Rozelle, Y. Shi, and L. Zhang (2014). Effect of migration on children's educational performance in rural China. *China's Economic Development*, 206–224.

Cherchye, L., B. De Rock, A. Lewbel, and F. Vermeulen (2015). Sharing rule identification for general collective consumption models. *Econometrica 83*(5), 2001–2041.

Cherchye, L., T. Demuynck, B. De Rock, and F. Vermeulen (2017). Household consumption when the marriage is stable. *American Economic Review 107*(6), 1507–1534.

Cherchye, L., B. D. Rock, and F. Vermeulen (2012). Married with children: A collective labor supply model with detailed time use and intrahousehold expenditure information. *American Economic Review 102*(7), 3377–3405.

Chiappori, P.-A. (1988). Rational household labor supply. *Econometrica*, 63–90.

Chiappori, P.-A. (1992). Collective labor supply and welfare. *Journal of Political Economy 100*(3), 437–467.

Chiappori, P.-A. and I. Ekeland (2009). The microeconomics of efficient group behavior: Identification. *Econometrica 77*(3), 763–799.

Chiappori, P.-A., B. Fortin, and G. Lacroix (2002). Marriage market, divorce legislation, and household labor supply. *Journal of Political Economy 110*(1), 37–72.

Chiappori, P.-A. and M. Mazzocco (2017). Static and intertemporal household decisions. *Journal of Economic Literature 55*(3), 985–1045.

Doepke, M. and M. Tertilt (2016). Asymmetric information in couples. *Unpublished Mimeo*.

Dunbar, G. R., A. Lewbel, and K. Pendakur (2013). Children's resources in collective households: Identification, estimation, and an application to child poverty in Malawi. *American Economic Review 103*(1), 438–471.

Fong, Y.-f. and J. Zhang (2001). The identification of unobservable independent and spousal leisure. *Journal of Political Economy 109*(1), 191–202.

Gibson, J. and D. McKenzie (2014). The development impact of a best practice seasonal worker policy. *Review of Economics and Statistics 96*(2), 229–243.

Gibson, J., D. McKenzie, and S. Stillman (2011). The impacts of international migration on remaining household members: Omnibus results from a migration lottery program. *Review of Economics and Statistics 93*(4), 1297–1318.

Institute of Social Science Survey (2015). *China Family Panel Studies (CFPS), https://doi.org/10.18170/DVN/45LCSO.* Peking University Open Research Data Platform, V42.

Kamenica, E. (2019). Bayesian persuasion and information design. *Annual Review of Economics 11*, 249–272.

Kamenica, E. and M. Gentzkow (2011). Bayesian persuasion. *American Economic Review 101*(6), 2590–2615.

Lechene, V., K. Pendakur, and A. Wolf (2022). Ordinary least squares estimation of the intrahousehold distribution of expenditure. *Journal of Political Economy 130*(3), 681–731.

Lewbel, A. and K. Pendakur (2008). Estimation of collective household models with engel curves. *Journal of Econometrics 147*(2), 350–358.

Lewbel, A. and K. Pendakur (2022). Inefficient collective households: Cooperation and consumption. *Economic Journal 132*(645), 1882–1893.

Lise, J. and S. Seitz (2011). Consumption inequality and intra-household allocations. *Review of Economic Studies 78*(1), 328–355.

Lu, Y. (2014). Parental migration and education of left-behind children: A comparison of two settings. *Journal of Marriage and Family 76*(5), 1082–1098.

Lyle, D. S. (2006). Using military deployments and job assignments to estimate the effect of parental absences and household relocations on children's academic achievement. *Journal of Labor Economics 24*(2), 319–350.

McKenzie, D. and H. Rapoport (2011). Can migration reduce educational attainment? evidence from Mexico. *Journal of Population Economics 24*, 1331–1358.

Menon, M., K. Pendakur, and F. Perali (2012). On the expenditure-dependence of children's resource shares. *Economics Letters 117*(3), 739–742.

Train, K. E. (2009). *Discrete choice methods with simulation*. Cambridge university press.

Trivers, R. L. (2017). Parental investment and sexual selection. In *Sexual Selection and the Descent of Man*, pp. 136–179. Routledge.

Vuong, Q. H. (1989). Likelihood ratio tests for model selection and non-nested hypotheses. *Econometrica*, 307–333.

Yue, A., Y. Bai, Y. Shi, R. Luo, S. Rozelle, A. Medina, and S. Sylvia (2020). Parental migration and early childhood development in rural China. *Demography 57*(2), 403–422.

Zhang, H., J. R. Behrman, C. S. Fan, X. Wei, and J. Zhang (2014). Does parental absence reduce cognitive achievements? evidence from rural China. *Journal of Development Economics 111*, 181–195.

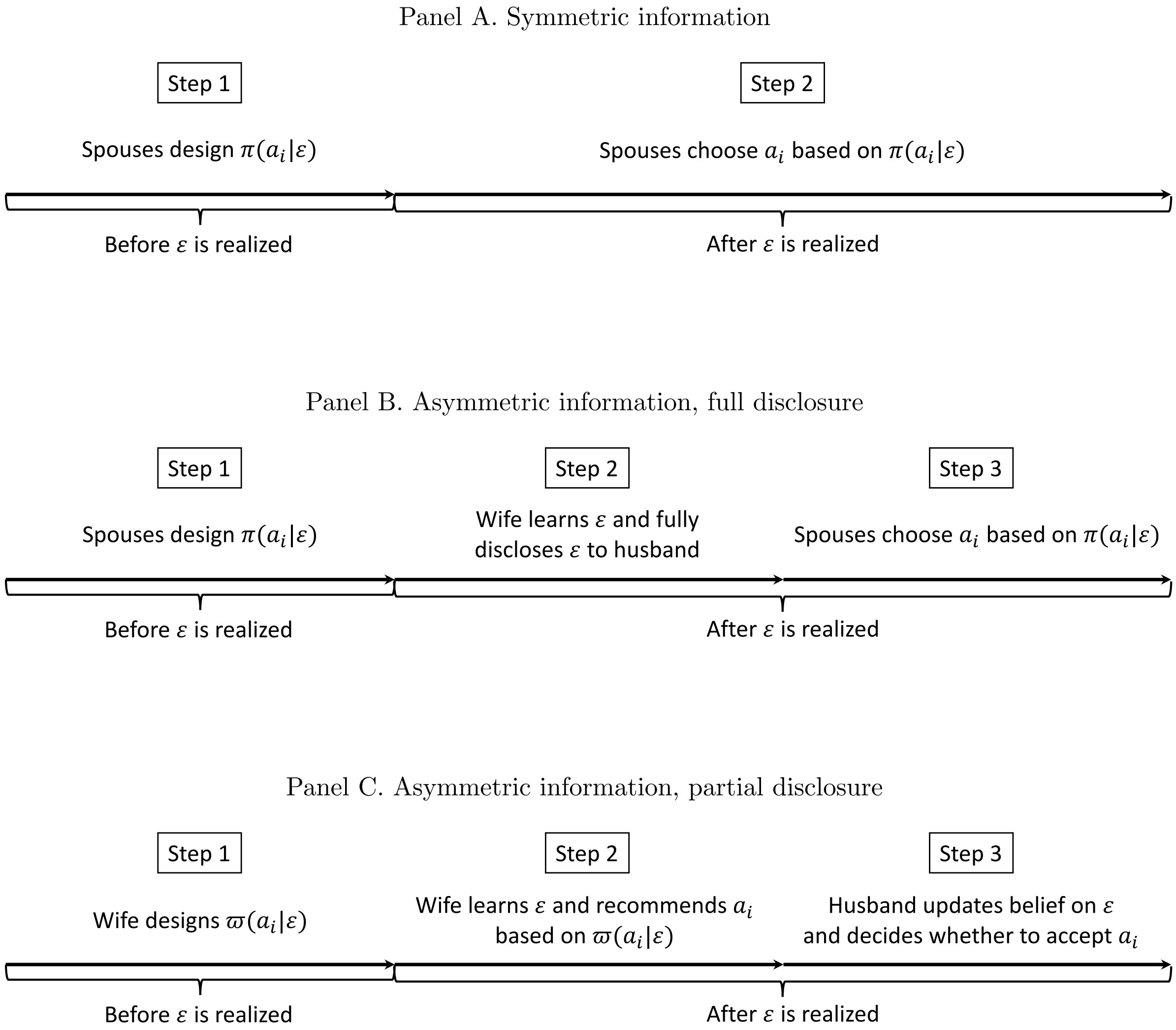

Figure 1 Decision process with symmetric and asymmetric information

Notes: Panel A plots the decision process in the case where both spouses learn the value of ε after its realization. Panels B and C plot the decision process in the case where wife learns the value of ε but husband remains uninformed, under full and partial information disclosure, respectively.

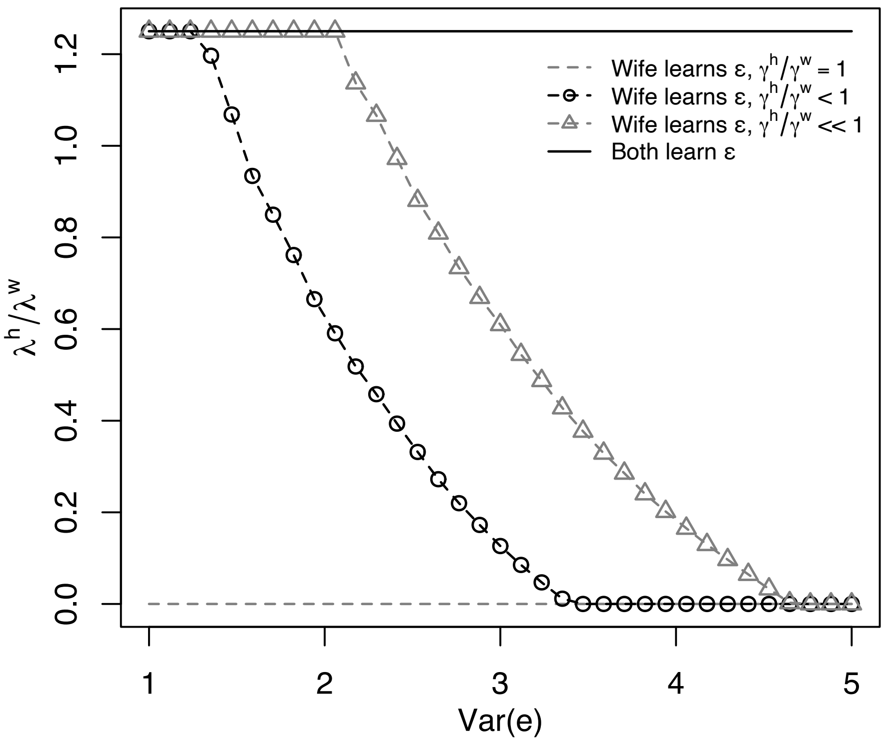

Figure 2 Relative bargaining power

Notes: This figure plots the relative bargaining power of husband. The dark solid line depicts the case where wife learns the value of e but husband is uninformed, with $\gamma^h < \gamma^w$ under partial information disclosure. The light solid line depicts the case with an even smaller γ^h. The dash-dotted line depicts the case where spouses' preferences are perfectly aligned, i.e. $\gamma^h = \gamma^w$. The dashed line depicts the case with symmetric information.

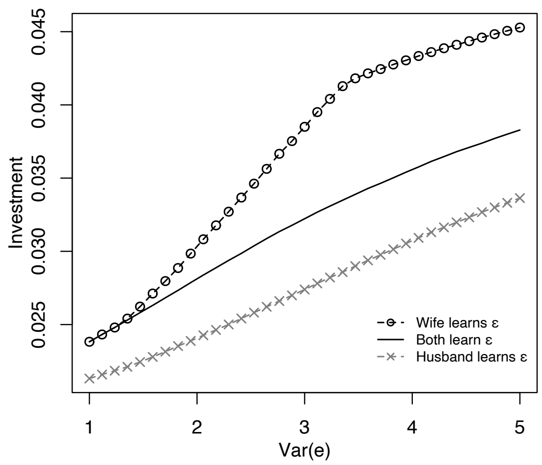

Figure 3 The level of investment in child

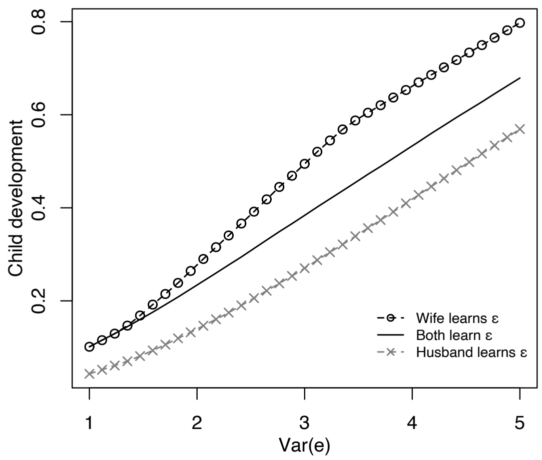

Figure 4 The level of child development

Notes: These two figures plot the level of investment and child development, focusing on the case with $\gamma^h < \gamma^w$. The dark solid line depicts the case where wife learns the value of e but husband is uninformed, under partial information disclosure. The dashed line depicts the case with symmetric information. The dotted line depicts the case where husband learns the value of e but wife is uninformed.

Table 1 Parental migration and child extracurricular educational expenditure

Dependent variable	Child extracurricular educational expenditure, log			
	Single children		First-born children	
	(1)	(2)	(3)	(4)
Only father migrates	0.632**	0.571**	0.483***	0.443***
	(0.271)	(0.276)	(0.146)	(0.152)
Only mother migrates	-0.228	-0.302	-0.165	-0.194
	(0.791)	(0.786)	(0.456)	(0.461)
Observations	732	732	1,429	1,429
R-squared	0.425	0.426	0.394	0.394
Baseline controls	Yes	Yes	Yes	Yes
Household total income, log		Yes		Yes
Number of children			Yes	Yes
County fixed effects	Yes	Yes	Yes	Yes

Notes: Data are from the 2010 CFPS survey. The sample comprises rural families with at least one child aged 6 to 15. The dependent variable is the log yearly extracurricular educational expenditure per child, which includes private tutoring fees, book expenses, and accommodation fees—but excludes formal education fees such as tuition. Families in which both parents migrate are excluded (less than two percent of the sample). All regressions incorporate controls for whether only the father migrates and whether only the mother migrates. Families with both parents staying at home serve as the reference group. The first two columns focus on one-child families. Column 1 includes baseline controls: the age and gender of the child, and the age and schooling years of both parents. Column 2 further includes log household income. The last two columns focus on first-born children. Column 3 includes baseline controls along with the number of children in the family. Column 4 further includes log household income. Standard errors given in parentheses are clustered at the county level.

***Significant at the 1 percent level.

**Significant at the 5 percent level.

*Significant at the 10 percent level.

Table 2 Estimation bias and the Vuong test with different error distribution assumptions

Distribution in data generating process			Normal			Logistic			Extreme		
Distribution assumed			Normal	Logistic	Extreme	Logistic	Normal	Extreme	Extreme	Normal	Logistic
Panel A. Var(e) = 2											
$(\gamma^w)^{-1}$	6.250	Bias	0.006	0.233	0.860	-0.013	-0.411	0.542	-0.009	-0.719	-0.585
		RMSE	0.284	0.513	0.932	0.322	0.498	0.668	0.412	0.772	0.680
$(\gamma^h)^{-1}$	12.50	Bias	-0.036	-0.420	-2.571	-0.007	0.717	-1.545	-0.012	1.682	0.855
		RMSE	0.441	0.629	2.639	0.303	0.802	1.626	0.378	1.709	0.897
λ_o^h/λ_o^w	1.250	Bias	-0.006	-0.418	-0.123	0.020	0.882	0.258	0.023	0.501	-0.048
		RMSE	0.182	0.425	0.165	0.132	0.940	0.299	0.152	0.583	0.157
λ^h/λ^w	–	Bias	0.002	-0.049	-0.019	0.115	0.112	0.100	0.001	0.018	0.016
		RMSE	0.044	0.074	0.051	0.118	0.118	0.106	0.014	0.024	0.023
Investment	–	Bias	0.000	-0.010	0.023	-0.004	0.006	0.029	-0.001	-0.025	-0.033
		RMSE	0.006	0.012	0.024	0.007	0.008	0.030	0.008	0.026	0.033
Child development	–	Bias	0.000	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		RMSE	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
-LL			4,140	4,152	4,159	4,039	4,049	4,045	4,023	4,043	4,030
Vuong test power			0.000	0.456	0.790	0.000	0.232	0.096	0.000	0.796	0.526
Panel B. Var(e) = 4											
$(\gamma^w)^{-1}$	3.125	Bias	-0.011	-0.628	-1.543	0.005	0.655	-0.786	0.006	1.281	0.642
		RMSE	0.250	0.670	1.567	0.248	0.705	0.841	0.312	1.308	0.692
$(\gamma^h)^{-1}$	6.250	Bias	-0.016	-0.371	-1.721	-0.014	0.373	-1.322	-0.008	1.619	1.124
		RMSE	0.224	0.422	1.736	0.211	0.441	1.343	0.271	1.638	1.148
λ_o^h/λ_o^w	1.250	Bias	0.004	-0.025	-0.062	0.012	0.052	-0.026	0.012	0.074	0.038
		RMSE	0.159	0.156	0.161	0.152	0.165	0.148	0.168	0.195	0.177
λ^h/λ^w	–	Bias	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
		RMSE	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000
Investment	–	Bias	-0.001	-0.013	0.008	-0.001	0.010	0.022	-0.001	-0.016	-0.026
		RMSE	0.008	0.015	0.012	0.008	0.013	0.024	0.010	0.019	0.028
Child development	–	Bias	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		RMSE	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
-LL			5,607	5,607	5,618	5,473	5,473	5,482	5,225	5,237	5,234
Vuong test power			0.000	0.164	0.588	0.000	0.110	0.602	0.000	0.606	0.538

Notes: The estimation method employed is maximum likelihood estimation. RMSE=Root mean square error. LL=Log likelihood. The last row in each panel reports the power of Vuong's (1989) test, where the null hypothesis is that the competing models are equally close to the true data generating process while the alternative is that one is closer.

Appendix

I Proofs in Section 2

I.1 Proposition 1

We first show that the optimal strategy is a cutoff strategy, such that $\pi^* (a_1 \mid e) = \mathbb{1}(e \geq c^*)$.

Suppose it is not, then two cases are possible: (i) the household chooses a_1 with a probability $\tau \in (0,1)$ for some e with a positive measure; (ii) there exists at least a cutoff c, such that the household chooses a_1 with probability 1 when the realized e is smaller than c, and a_2 otherwise. For simplicity, we restrict our statement to two intervals: $(c_1, c_2]$ and (c_2, c_3), where $-\infty \leq c_1 < c_2 < c_3 \leq \infty$. The following decision strategy $\pi^0(a_1 \mid e)$ incorporates the two cases:

$$\pi^0(a_1 \mid e) = \begin{cases} \tau_1 \in (0,1] & \text{if } e \in (c_1, c_2]\,, \\ \tau_2 \in [0,1) & \text{if } e \in (c_2, c_3)\,. \end{cases} \tag{A1}$$

Suppose $\pi^0(a_1 \mid e)$ in equation A1 (rather than a cutoff strategy) is the optimal strategy, the household obtains the expected value:

$$V(\pi^0) = \int_{c_1}^{c_2} \tau_1 \; v(e) \; \mathrm{d}G(e) + \int_{c_2}^{c_3} \tau_2 \; v(e) \; \mathrm{d}G(e) + \overline{V}^*, \tag{A2}$$

where $v(e) = \lambda^h v^h + v^w + (\lambda^h + 1)e$, and $\overline{V}^*$ represents the expected value for $e \notin (c_1, c_3)$.

As the cumulative distribution function $G(\cdot)$ increases with e, for some $c' \in (c_1, c_2)$, there must exist a $\tau' > \tau_2$ such that

$$\begin{aligned} &\tau_1 \left(G\left(c_2\right) - G(c')\right) + \tau' \left(G\left(c_3\right) - G\left(c_2\right)\right) = \tau_1 \left(G\left(c_2\right) - G\left(c_1\right)\right) + \tau_2 \left(G\left(c_3\right) - G\left(c_2\right)\right) \\ \Longleftrightarrow &\tau_1 \left(G(c') - G\left(c_1\right)\right) = \left(\tau' - \tau_2\right) \left(G\left(c_3\right) - G\left(c_2\right)\right). \end{aligned} \tag{A3}$$

If there exists another strategy $\pi'(a_1 \mid e)$, with which the expected value is strictly higher than $V(\pi^0)$ in equation A2, this generates a contradiction. Consider the following strategy:

$$\pi'(a_1 \mid e) = \begin{cases} 0 & \text{if } e \in (c_1, c']\,, \\ \tau_1 \in (0,1] & \text{if } e \in (c', c_2]\,, \\ \tau' \in (0,1] & \text{if } e \in (c_2, c_3)\,, \end{cases} \tag{A4}$$

where c' and τ' are defined in equation A3.

We then show that $V(\pi^0) < V(\pi')$. Given $c_1 < c' < c_2$, $v(e)$ increasing with e, and equation

A3, we have:

$$
\begin{aligned}
&\int_{c_1}^{c_2} \tau_1 \cdot v(e) \mathrm{d}G(e) + \int_{c_2}^{c_3} \tau_2 \cdot v(e) \mathrm{d}G(e) \\
=&\tau_1 \left(G\left(c_2\right) - G\left(c_1\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_1 < e \leq c_2\right) + \tau_2 \left(G\left(c_3\right) - G\left(c_2\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right) \\
<&\tau_1 \left(G\left(c_2\right) - G\left(c_1\right) \cdot \mathbb{E}\left(v(e) \mid c' < e \leq c_2\right) + \tau_2 \left(G\left(c_3\right) - G\left(c_2\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right) \\
=&\tau_1 \left(G\left(c_2\right) - G(c')\right) \cdot \mathbb{E}\left(v(e) \mid c' < e \leq c_2\right) + \tau' \left(G\left(c_3\right) - G\left(c_2\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right) \\
&+ \tau_1 \left(G(c') - G\left(c_1\right)\right) \cdot \mathbb{E}\left(v(e) \mid c' < e \leq c_2\right) \\
&- \left(\tau' - \tau_2\right) \left(G\left(c_3\right) - G\left(c_2\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right) \\
=&\tau_1 \left(G\left(c_2\right) - G(c')\right) \cdot \mathbb{E}\left(v(e) \mid c' < e \leq c_2\right) + \tau' \left(G\left(c_3\right) - G\left(c_2\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right) \\
&+ \underbrace{\tau_1 \left(G(c') - G\left(c_1\right)\right) \cdot \left[\mathbb{E}\left(v(e) \mid c' < e \leq c_2\right) - \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right)\right]}_{<0} \\
<&\tau_1 \left(G\left(c_2\right) - G(c')\right) \cdot \mathbb{E}\left(v(e) \mid c' < e \leq c_2\right) + \tau' \left(G\left(c_3\right) - G\left(c_2\right)\right) \cdot \mathbb{E}\left(v(e) \mid c_2 < e < c_3\right) \\
=&\int_{c'}^{c_2} \tau_1 \cdot v(e) \mathrm{d}G(e) + \int_{c_2}^{c_3} \tau' \cdot v(e) \mathrm{d}G(e).
\end{aligned} \tag{A5}
$$

It follows that:

$$
\begin{aligned}
V(\pi^0) =& \int_{c_1}^{c_2} \tau_1 \cdot v(e) \mathrm{d}G(e) + \int_{c_2}^{c_3} \tau_2 \cdot v(e) \mathrm{d}G(e) + \overline{V}^* \\
<& \int_{c'}^{c_2} \tau_1 \cdot v(e) \mathrm{d}G(e) + \int_{c_2}^{c_3} \tau' \cdot v(e) \mathrm{d}G(e) + \overline{V}^* \\
=& V(\pi').
\end{aligned} \tag{A6}
$$

The result that $V(\pi^0) < V(\pi')$ contradicts $\pi^0(a_1 \mid e)$ being the optimal strategy. That is, the optimal strategy is a cutoff strategy, such that $\pi^*\left(a_1 \mid e\right) = \mathbb{1}(e \geq c^*)$.

So our problem is to solve:

$$
\max_c \int_c^{\infty} v(e) \; \mathrm{d}G(e), \tag{A7}
$$

such that

$$
\begin{aligned}
& \lambda^h v^h + v^w + (\lambda^h + 1) \; c^* = 0 \\
\Longrightarrow & c^* = -\frac{\lambda^h v^h + v^w}{\lambda^h + 1}.
\end{aligned} \tag{A8}
$$

I.2 Lemma 1

Following equation 7 in the main text, we have:

$$\begin{aligned} &\int (\nu_1^h + \varepsilon_1)\ \mathrm{d}F_{\varpi}(\varepsilon \mid a_1) \geq \int (\nu_2^h + \varepsilon_2)\ \mathrm{d}F_{\varpi}(\varepsilon \mid a_1), \\ \Longrightarrow &\int (v^h + e)\ \mathrm{d}F_{\varpi}(e \mid a_1) \geq 0, \\ \Longrightarrow &\int \pi(a_1 \mid e)\ (v^h + e)\ \mathrm{d}G(e) \geq 0. \end{aligned} \tag{A9}$$

Similarly,

$$\begin{aligned} &\int (\nu_2^h + \varepsilon_2)\ \mathrm{d}F_{\varpi}(\varepsilon \mid a_2) \geq \int (\nu_1^h + \varepsilon_1)\ \mathrm{d}F_{\varpi}(\varepsilon \mid a_2), \\ \Longrightarrow &0 \geq \int \pi(a_2 \mid e)\ (v^h + e)\ \mathrm{d}G(e), \\ \Longrightarrow &v^h \leq v^h - \int \pi(a_2 \mid e)\ (v^h + e)\ \mathrm{d}G(e), \\ \Longrightarrow &\int \pi(a_1 \mid e)\ (v^h + e)\ \mathrm{d}G(e) \geq v^h. \end{aligned} \tag{A10}$$

Based on equations A9 and A10, we have:

$$\int \pi(a_1 \mid e)\ (v^h + e)\ \mathrm{d}G(e) \geq \max\{v^h, 0\}. \tag{A11}$$

I.3 Proposition 2

When $u_o^h \leq \max\{v^h, 0\}$, the result follows immediately from Proposition 1.

When $u_o^h > \max\{v^h, 0\}$, given that $\pi^*(a_1 \mid e) = \mathbb{1}(e \geq c^*)$, our problem becomes:

$$\begin{aligned} &\max_c \int_c^{\infty} (v^w + e)\ \mathrm{d}G(e) \\ &\text{s.t.} \int_c^{\infty} (v^h + e)\ \mathrm{d}G(e) \geq \max\{v^h, 0\}. \end{aligned} \tag{A12}$$

Let λ^h be the Karush-Kuhn-Tucker multipliers for the constraint. The Lagrangian function is then given by:

$$L = V^w(c) + \lambda^h \left(V^h(c) - \max\{v^h, 0\}\right), \tag{A13}$$

where $V^m(c) = \int_c^{\infty} (v^m + e)\ \mathrm{d}G(e)$ for $m \in \{h, w\}$.

Let c^* be an optimal cutoff, then the first order conditions for optimization are:

$$\begin{cases} \dfrac{\partial L}{\partial c} = \dfrac{\partial V^w(c^*)}{\partial c} + \lambda^{h*} \dfrac{\partial V^h(c^*)}{\partial c} = 0, \\ \dfrac{\partial L}{\partial \lambda^h} = V^h(c^*) \geq 0\ ,\ \lambda^{h*} \geq 0\ ,\ \lambda^{h*} \dfrac{\partial L}{\partial \lambda^h} = 0. \end{cases} \tag{A14}$$

Case 1: interior solution $\lambda^{h*} = 0$.

$$\begin{aligned}\frac{\partial L}{\partial c} &= \frac{\partial V^w(c^*)}{\partial c} \\ &= -(v^w + c^*)\ g(c^*) \\ &= 0.\end{aligned} \tag{A15}$$

Then we have

$$c^* = -v^w. \tag{A16}$$

Case 2: corner solution $\lambda^{h*} > 0$.

$$\frac{\partial L}{\partial \lambda^h} = V^h(c^*) = \max\{v^h, 0\}. \tag{A17}$$

When $v^h > 0$ we have

$$c^* = k(-v^h), \tag{A18}$$

where $k^{-1}(c) = \mathbb{E}\left[e \mid e < c\right]$; and when $v^h < 0$ we have

$$c^* = q(-v^h), \tag{A19}$$

where $q^{-1}(c) = \mathbb{E}\left[e \mid e \geq c\right]$.

Therefore, when $u_o^h > \max\{v^h, 0\}$,

$$c^* = \begin{cases} k(-v^h) & \text{if } \ v^h > 0 \text{ and } \ k(-v^h) < -v^w, \\ q(-v^h) & \text{if } \ v^h < 0 \text{ and } \ q(-v^h) > -v^w, \\ -v^w & \text{otherwise.} \end{cases} \tag{A20}$$

I.4 Proposition 3

The result follows immediately from Lemma 1 and Proposition 2.

II Proofs in Section 4

II.1 Proposition 4

The model with multiple choices can be decomposed into C_I^2 problems with binary choices, where I is the number of choices. Then the result follows immediately from Proposition 1.

II.2 Proposition 5

The model with multiple choices can be decomposed into C_I^2 problems with binary choices, where I is the number of choices. Then the result follows immediately from Proposition 2.

II.3 Proposition 6

With partial information disclosure, the Lagrange multiplier for constraint in equation 23 in the main text is:

$$\lambda_{ij}^{h*} = -\frac{v_{ij}^w + c_{ij}^*}{v_{ij}^h + c_{ij}^*}, \tag{A21}$$

where c_{ij}^* is given by equation 24. With Assumption 2, we have

$$k_{ij}(-v_{ij}^h) = -v_{ij}^h \ \tilde{q}(1), \tag{A22}$$

$$q_{ij}(-v_{ij}^h) = -v_{ij}^h \ \tilde{q}(1). \tag{A23}$$

Then when c_{ij}^* equals $k_{ij}(-v_{ij}^h)$ or $q_{ij}(-v_{ij}^h)$, we have

$$\begin{aligned}\lambda_{ij}^{h*} &= -\frac{bv_{ij}^h - v_{ij}^h \ \tilde{q}(1)}{v_{ij}^h - v_{ij}^h \ \tilde{q}(1)} \\ &= -\frac{b - \tilde{q}(1)}{1 - \tilde{q}(1)}, \text{ for } \forall i, j \leq I, i \neq j \\ &= \lambda^{h*}.\end{aligned} \tag{A24}$$

When c_{ij}^* equals $-v_{ij}^w$, it is straightforward that $\lambda^{h*} = 0$. Therefore, we have

$$\lambda^{h*} = \max\left\{0, \ -\frac{b - \tilde{q}(1)}{1 - \tilde{q}(1)}\right\}. \tag{A25}$$

II.4 Proposition 7

The result follows immediately from Propositions 1 and 4.

II.5 Proposition 8

The result follows immediately from Propositions 2 and 5.

II.6 Proposition 9

Under partial information disclosure, the Lagrange multiplier for constraint in equation 30 is

$$\frac{\lambda_{ij}^{m*}}{\lambda_o^m} = -\frac{\sum_{m\in S} \lambda_o^m \ v_{ij}^m + \sum_{m\in S} \lambda_o^m \ c_{ij}^*}{\sum_{m\in R} \lambda_o^m \ v_{ij}^m + \sum_{m\in R} \lambda_o^m \ c_{ij}^*}, \tag{A26}$$

where c^*_{ij} is given by equation 31. With Assumption 2', we have

$$k_{ij}(-\bar{v}^R_{ij}) = -v^{y_0}_{ij} \cdot \tilde{q}\left(\bar{b}^R\right), \tag{A27}$$

$$q_{ij}(-\bar{v}^R_{ij}) = -v^{y_0}_{ij} \cdot \tilde{q}\left(\bar{b}^R\right), \tag{A28}$$

where y_0 is a random fixed player. Then when c^*_{ij} equals $k_{ij}\left(-\bar{v}^R_{ij}\right)$ or $q_{ij}\left(-\bar{v}^R_{ij}\right)$, we have

$$\begin{aligned} \lambda^{m*}_{ij} &= -\lambda^m_o \frac{b^S - \sum_{m\in S} \lambda^m_o\ \tilde{q}(\bar{b}^R)}{b^R - \sum_{m\in R} \lambda^m_o\ \tilde{q}(\bar{b}^R)},\ \text{for } \forall i,j \leq I, i \neq j \\ &= \lambda^{m*}. \end{aligned} \tag{A29}$$

When c^*_{ij} equals $-\bar{v}^S_{ij}$, it is straightforward that $\lambda^{m*} = 0$.