Fire Sales and Endogenous Volatility

Yue Jiang
Boston University

Draft Version, September 2016

Abstract

After the collapse of the housing bubble in 2007, severe fire sales of assets in the financial sector are accompanied by a rise in the volatility of asset returns in the non-financial firms. To account for their co-movements, I develop a model that highlights the interaction between the financial health of the banking sector and the volatility of asset returns. The novel feature of the model is that the volatility of asset returns is endogenously generated by the banks' risk taking behavior. The risk taking by banks imposes a negative externality on the financial health of other banks because given the risk aversion of secondary market buyers, the liquidation of risky assets depresses the secondary market price of assets. A weak financial health hurts the bank's long term profitability. Combining with the limited liability, the model can give rise to a vicious feedback loop between a collective risk taking behavior in the banking sector and fire sales of assets. A standard liquidity requirement is shown to have ambiguous effects in stabilizing the financial system depending on the asset market liquidity. The model suggests a room for counter-cyclical macro-prudential policy to improve financial stability.

[1] I am deeply indebted to Christophe Chamley for his constant support and advice during this project. I also benefitted greatly from Dirk Hackbarth, Jianjun Miao, Simon Gilchrist, Stephen Terry and Anton Korinek. Comments are welcome. Email address: yuej@bu.edu.

1 Introduction

The 2007 financial crisis has rekindled the search of the origins of financial fragility. The collapse of the housing bubble in 2007 triggered a liquidity shortage in the banking sector.[1] Banks' financial health deteriorated due to massive losses on assets and withdrawals from short term creditors, which forced banks to liquidate assets in fire sales.[2] Along with the severe fire sales of assets, the economy also experienced a widespread rise in the volatility of asset returns among non-financial firms. Figure 1 documents the evolution of the time-varying volatility of equity returns for non-financial firms[3] and the TED spread[4] in the U.S.. The TED spread indicates the difficulty of the banking sector in financing their long term investments and can be used as a measure of the credit risk. Both variables surged significantly during the recent recession. Moreover, their co-movement seems persistent throughout the past 30 years. To account for these observations, I provide a theoretical explanation that highlights the interaction between the financial health of the banking sector and the volatility of asset returns to explain their co-movement and to explore the potential cause of financial instability.

The novel feature of the model is that the volatility of asset returns is endogenously determined by banks' risk taking behavior. The bank invests its funds in a project that yield returns in the long run. The bank is able to choose risk on its long term assets by investing in a specific project among a menu of projects differing in their riskiness. A risky investment results in a high volatility in asset returns.

I incorporate the bank's choice of risk in a model, the framework of which is borrowed from Diamond and Dybvig(1983). Banks choose risk on their long term investments, expecting that some unknown fraction of depositors will withdraw early. If they face a high demand for funds, banks are forced to liquidate their long term assets prematurely in the secondary market at some endogenous price for their assets.

The choice of risk poses a tradeoff for banks. On the one hand, subject to limited liability, the bank can benefit from the option to default. On the other hand, the bank incurs a cost for monitoring and collecting the risky returns[5]. The incentive to risk shifting[6] is stronger when the bank is close to default. Banks are closer to default when

[1] See Brunnermeier (2009) for a detailed description.

[2] The term "fire sale", coined by Shleifer and Vishny (1992), is defined as a forced sale of an asset at a dislocated price. See Shleifer and Vishny (2011) and French et al (2010).

[3] See the appendix for the details of the calculation.

[4] The TED spread is calculated as the spread between the 3-month LIBOR and the 3-month Treasury bill. It is collected from FRED.

[5] as in Holmstrom and Tirole(1997)

[6] The risk shifting behavior is an investment strategy that increases the probability of losses, if it turns out badly, the creditors bear most of the costs, whereas if it turns out well, it gives the shareholders most of the gains. See Jensen and Meckling (1976)

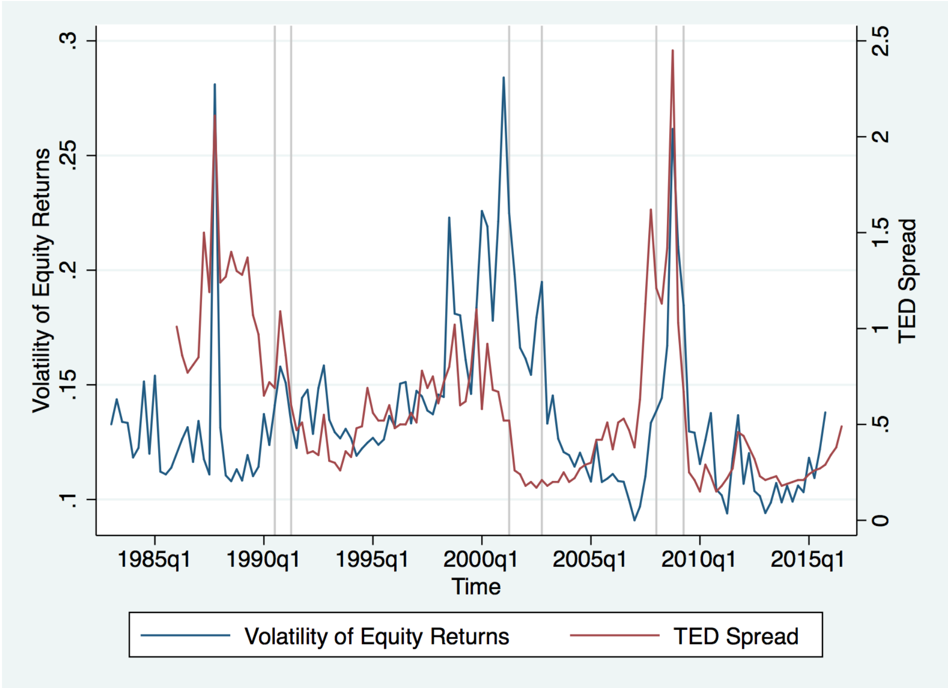

Figure 1

the secondary market price for assets is low. This so-called "gamble for resurrection" effect is shown to exist in real life (e.g., Eisdorfer (2008), Baldursson and Portes (2013)).[7]

The key mechanism in the model is the following. The risk taking by banks imposes a negative externality on the financial health of other banks because given the risk aversion of buyers in the secondary market, liquidation of risky asset generates a downward pressure on the secondary market price. A low secondary market price inhibits these banks' capability of providing liquidity to their depositors, leading to a loss in their long term profitability. With limited liability, banks have stronger incentive to exploit the option value of default by taking risk. Therefore, the risk taking by banks generates strategic complementarity in this model.

Because of the strategic complementarity of risk taking, the model can give rise to multiple equilibria. There exists an equilibrium with low volatility in asset returns, low default probability of banks, and a high secondary market price. The low volatility equilibirium represents the economy in normal times. There potentially exists another equilibrium with high return volatility, high credit risk and a low price in the secondary

[7]Eisdorfer (2008) provides empirical evidence of risk-shifting behavior among financially distressed firms. Baldursson and Portes (2013) examine the "gambling for resurrection" behavior of banks in Iceland that drives the final collapse of the major banks in October 2008. It also has been reported that the U.S. banks increases sales of insurance against credit losses to holders of Greek, Portuguese, Irish, Spanish and Italian debt in the first half of 2011, boosting the payoff by raising the risk of it. http://www.bloomberg.com/news/articles/2011-11-01/selling-more-insurance-on-shaky-european-debt-raises-risk-for-u-s-banks

market. This riskier equilibrium characterizes the economy in financial crises, e.g., the 2007 financial crisis.

The equilibrium with a self-fulfilling crisis generates welfare loss because banks do not internalize their impact on asset prices or the default costs. Therefore, in the second part of the paper I analyze how macro-prudential policies affect the financial market efficiency in my setting. To be more specific, I extend the model by incorporating an ex-ante choice of cash holding and analyze the implication of a liquidity requirement, according to which banks are required to hold certain amount of liquid assets ex ante.

The model suggests that the effect of a liquidity requirement is ambiguous in improving financial stability. When the secondary market price is low, banks are holding liquid assets at a level which is lower than the social optimal level. Liquidity requirement could lower bank's incentive of risk shifting by reducing the credit risk and boosting bank payoffs. When the secondary market price is high, the cost of forfeited long term returns outweighs the benefit from a stronger liquidity buffer. Liquidity holdings reduce long term payoffs, leading to a stronger risk shifting incentive. Therefore, the liquidity requirement poses tradeoff between improving financial stability in economic downturns and encouraging excessive risk taking in economic upturns.

There has been a growing consensus on the implementation of counter-cyclical regulations in promoting the resilience of the financial system. In the context of the model, the counter-cyclical liquidity requirement[8] is shown to improve the tradeoff of a standard liquidity requirement and promote financial stability. The intuition is the following. According to the previous discussion, a higher liquidity requirement could effectively rein in excessive risk taking when fire sales are expected. However, it would not encourage risk taking by much during economic booms because the bank payoffs are less affected. Similarly, a lower liquidity requirement could reduce the risk taking incentives when the secondary market price is high while it may not significantly intensify the risk taking incentive because bank payoffs are already low during economic downturns.

To sum up, I build a model that connects financial health of banks with macroeconomic volatility. An expectation of fire sales can result in a self-fulfilling financial crisis where the risk taking incentives and fire sales reinforce each other. The model suggests a room for counter-cyclical macro-prudential policy to improve financial stability.

The paper is organized as follows. Section 2 discusses research related to the paper. Section 3 studies the basic version of the model, followed by an analysis of comparative statics. Section 4 extends the basic model by incorporating ex ante choice of liquidity and studies the implication of a liquidity requirement. Section 5 concludes. The Appendix

[8] A counter-cyclical liquidity requirement stipulates a higher requirement during economic upturns and a lower requirement in economic downturns.

provides the proofs.

2 Literature Review

The paper connects with several lines of the literature: (i) financial fragility; (ii) risk taking of banks; (iii) fire sales and (iv) time-varying volatility. I discuss how my paper is related to each of the topics and the papers in the intersection of these topics.

First, this paper is related to the literature on the financial fragility. In the seminal paper by Diamond and Dybvig (1983), the fragility of a financial system stems from "panics" of depositors on the amount of withdrawals. Along this line of work, Chari and Jagannathan (1988) show that bank runs occur not only when the economic outlook is poor but when liquidity needs are high as well. Allen and Gale (1998) develop a model where panics occur when depositors perceive that the returns on bank assets are going to be unusually low. Morris and Shin (2009) further study the panic-based bank run by relating such panic with economic fundamentals under a global-game framework.

This paper adopts some general structures from Diamond and Dybvig (1983) but constructs a distinct mechanism from theirs. In this paper, the risk taking of banks endangers the stability of the financial system by rendering the system more vulnerable to "panics" on fire sales in the secondary market.

An interesting paper that also points to the asset price in the secondary market as a key in generating financial instability is Malherbe (2014). Malherbe (2014) emphasizes the negative externality of cash hoarding. With asymmetric information about project qualities, cash hoarding triggers adverse selection in the secondary market, where banks only liquidate lemon projects.[9] The negative externality in my model is originated from a different source, the risk taking of banks. The risk taking of banks depresses the liquidation value of the assets, leading to a deteriorated financial health of banks.

Two papers have different implications on liquidity regulations. Malherbe (2014) renders the liquidity requirement counterproductive because of the negative externality of cash hoarding. In my paper, the effect of a liquidity requirement is ambiguous. The liquidity requirement encourages excessive risk taking by restraining banks' long term investments when the asset market is liquid. However, the liquidity requirement can help improving financial stability when the asset market is illiquid by lowering the credit risk of the banking sector.

Second, the paper is related to the literature on the risk taking of banks. Following the seminal papers by Jensen and Meckling (1976) (in a corporate finance context) and Stiglitz

[9] Dong, Miao and Wang (2015) considers the adverse selection mechanism in a dynamic general equilibrium model and emphasizes that funding liquidity can erode market liquidity.

and Weiss (1981) (in a credit market equilibrium context), the risk-shifting phenomenon has been intensively studied.[10] Acharya (2009) develops a model that highlights a systemic risk-shifting incentive that is originated from bank failures. When one bank fails, it exerts negative externality on others by raising the deposit rate. My paper differs from Acharya(2009) in two aspects. First, my paper does not rely on the actual defaults for the existence of a systemic risk-shifting incentive. Second, instead of choosing the correlation on their long term investments, banks in my model choose the volatility on asset returns. From this perspective, my paper complements Acharya (2009) by looking at the risk from another dimension, with a focus on the volatility in returns.

The way I model the risk taking of banks is similar to Martinez-Miera and Repullo (2015), Repullo(2004), and Navarro (2015). However, the paper focuses on the study of financial stability, which stems from the bank's expectation of future liquidity shortages and its inability of funding its depositors.

In the empirical front, there are works confirming the existence of the risk shifting incentives. Duran and Lozano-Vivas (2014) examine the risk shifting of US banks in 1998 - 2011. Their results suggest that the risk shifting is present throughout the entire period, with least significance for the post crisis period. Moreover, banks engage in risk shifting most significantly with non-depository creditors. Landier, Sraer and Thesmar (2011) conduct an interesting case study on the lending behavior of a large subprime mortgage originator - New Century Financial Corporation and show that financial institutions in distress, may take excessive risk. The incentive distortion effect of financial distress and gamble "for resurrection" of banks are intensively studied in Esty (1997), Gan(2004) and Fischer et al (2011). My paper provides theoretical explanation for these findings that banks in distress tend to engage in risk-shifting and the risk is mostly shifted to un-insured creditors.

Third, the paper is related to the literature on fire sales[11]. In light of the recent financial crisis, people have been focusing on the deterioration of balance sheets of banks and the disruptions of the so-called bank lending channel (See Bernanke and Blinder (1988)). Papers exploring the importance of financial intermediations include Gertler and Karadi (2011), Gertler and Kiyotaki (2009), Brunnermeier and Pedersen (2009), and Brunnermeier and Sannikov (2011).

Most papers in the literature focus on the contraction of credit supplies due to lower net worths of banks. Departing from the literature, this paper provides a new channel through which fire sales causes disruptions in the financial system by emphasizing the interaction between fires sales and the risk taking by banks.

[10]See Bhattacharya et al. (1998) and Freixas and Rochet (2008) for surveys on risk shifting.

[11]See Shleifer and Vishny (2011) for a survey on fire sales

Last, the paper is related to the literature on time-varying uncertainty. The seminal paper by Bloom(2009) points out that the time varying volatility can undermine the real economy.[12] Because of the detrimental effect of time-varying volatilities on the real economy, it is worth exploring where the volatility comes from.

There is a growing line of work that studies endogenous volatilities. A seminal paper on the study of endogenous volatility is Veldkamp (2005) where uncertainty is generated by learning about economic fundamentals.[13] Bachmann and Bayer (2013) use a long panel of German firms and show that shocks to the variance of firm-level TFP innovations, if any, only mildly amplify first-moment aggregate shocks. The volatility in TFP is not an independent source of aggregate fluctuations.[14] Bachmann and Moscarini(2012) explore the reverse causality where negative first moment shocks induce risky behavior, leading to a rise in volatility in economic outcomes.

Following this lead, I endogenize the volatility of asset returns by relating it with the risk taking by banks. Establishing the link between the volatility and the financial health of banks, the paper highlights the negative impact of volatility on the financial system and the real economy.

3 The Model

The model adopts the framework of Diamond-Dybvig Banking model[15]. There are three dates ($t = 0, 1, 2$). The key actors in the model are banks and depositors. Departing from the Diamond-Dybvig model, the paper focuses on the choice of risk by banks and incorporates a secondary market for the long term assets at date 1.

The main subject of study is the bank. However, the mechanism discussed in the paper is not limited to the financial institutions. It can be extended to non-financial corporations that finance their projects by short term debts. Exposed to the mismatched maturity, agents are prone to financial distress, which induces distortion in risk taking incentives.

The time-line can be summarized in Figure 2. The key elements in the timeline are the following: at date 0, the bank makes long term investments using the funds it attracts from depositors. The bank chooses the return risk on its long term assets given the mean

[12] A more recent work by Bloom, Floetotto, Saporta-Eksten, and Terry (2012) quantifies the effect of uncertainty shocks in a DSGE model with heterogeneous firms. Gilchrist, Sim and Zakrajsek(2013) incorporate default risk in a model with firm investment and explore the interaction between uncertainty shock and financial frictions.

[13] The mechanism has been extended in Orlik and Veldkamp (2014).

[14] Bachmann, Elstner and Sims (2010) form a more genuine measure of uncertainty and find that the shocks to uncertainty have no discernible impact on the aggregate economic activity.

[15] See Diamond and Dybvig (1983) for details

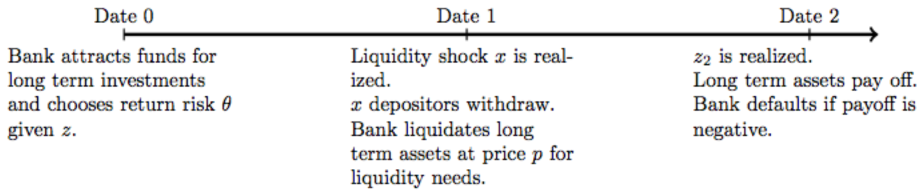

Figure 2: Time Line of the Model

return of the assets. At date 1, when its depositors demand funds, the bank needs to liquidate its long term assets in a secondary market to fulfill their needs. At date 2, long term assets pays off. Given limited liability, the bank has the option to default when its payoff is negative.

Long Term Assets Each bank invests in projects that yield returns at date 2. The projects have mean return z, which is realized at date 0. Because the bank makes decisions after the realization of z, in the basic model, z is treated as a parameter. Given the mean return z, the long term project generates random returns z_2 at date 2 according to

$$z_2(z, s; \theta) = (1 + s\theta)z. \tag{1}$$

In this equation, s is an idiosyncratic productivity shock at date 2,

$$s = \begin{cases} 1 & \text{with probability } \frac{1}{2}, \\ -1 & o.w. \end{cases} \tag{2}$$

θ represents the riskiness of returns. It is an endogenous choice of the bank. There is a menu of projects available for banks. These projects have the same mean returns but they differ in terms of riskiness. $\theta \in [0, \bar{\theta}]$. For simplicity, I assume $\bar{\theta} = 1$. The bank chooses risk θ by investing in a specific project.

Banks There is a measure one of ex ante identical banks indexed by i. $i \in [0, 1]$. At date 0, each bank attracts one unit of deposits from households and uses the funds to invest in long term assets. In the basic model, banks cannot hold liquid assets ex ante.[16] The deposits are uninsured and generate return R^* at date 2.[17]

[16]Later on, the paper allows banks to invest in liquid assets that yield risk free returns the next period.

[17]In reality, deposits are insured or regulated in many countries. In the U.S, FDIC has been created in 1933 to provide deposit insurance to depositors in US banks. Apart from commercial banks that are FDIC-insured, there are other non-FDIC-insured financial corporations, such as investment banks and

Given the mean return of long term assets z, the bank chooses to take risk θ on its portfolio. There is a cost of risk taking $\kappa(\theta)$ for each unit of return. Intuitively, the cost can be interpreted as the resources spent for to monitor and collect the realized returns. Assume that $\kappa(\theta) = c\theta$, where $c < \frac{1}{2}$. [18]

At date 1, a common liquidity shock[19] hits all banks with probability λ, in which a fraction x of their depositors withdraw. x is the realization of a random variable drawn from an uniform distribution on $[0, 1]$. x is private information to the bank and it is not observable to its depositors.

Each bank needs to offer x liquid assets to satisfy the early withdrawals. At the same time, the bank offers late depositors a return R^* at date 2 so that they have no incentive to withdraw at date 1.

In order to pay the early withdrawals, the bank has to liquidate its long term assets in the secondary market at price p. p is the price facing all selling banks. In the basic model without ex ante holding of liquid assets, p is also the market value of bank assets at date 1. If $p > x$, the bank is fine. If $p < x$, the bank cannot cover the withdrawals even by selling all long term assets. In this case, the bank is forced to default. Let

$$\psi(p) = \min\{p, 1\}. \tag{3}$$

If a shock takes place, $1 - \psi(p)$ is the probability that a bank faces withdrawal x greater than p. $\lambda(1 - \psi(p))$ is the probability (in date 0) of default (in date 1). Call it the *'illiquidity risk'* of the bank.

In this model, the instability of the financial system stems from the early withdrawal shocks or more precisely, the expectation of the shock. In expectation of an early withdrawal shock, the bank chooses risk θ to maximize its long term payoff. The aggregate riskiness of asset returns determines the asset price in secondary market, which in turn affects the risk decision of banks through its impact on the illiquidity risk and the credit risk of the bank.

At date 2, the productivity shock s is realized. The long term assets pay off accordingly. The ex post payoff of the bank is

$$y(z, x, s; \theta) = (1 - \frac{x}{p})(1 - \kappa(\theta))(1 + s\theta)z - (1 - x)R^*. \tag{4}$$

With limited liability, a bank will default whenever its payoff is negative. When default,

funds, that finance their long term investments with short term debts. The model is more relevant to this type of financial institutions.

[18] The assumption on c guarantees that the per unit return ex ante with risk is greater than the return with no risk.

[19] The same type of shock has also been used in Diamond and Rajan (2011).

the bank will get zero payoff.

Depositors Each bank has a measure 1 of ex ante identical depositors. At date 1, with probability λ, x fraction of depositors become the early type, which need to withdraw funds immediately. The return for early withdrawals is 1. $(1-x)$ depositors become the late types, who have no needs for funds at date 1.

The late type depositors have the option to withdraw fund together with the early types and invest in safe and liquid assets which yield risk-free return $\bar{r}$ at date 2. $\bar{r}$ is exogenous. If the late types do not withdraw, they will get long term return R^* at date 2 given that the bank does not default. Otherwise, they will get 0 when the bank defaults.[20] The mean return of long term asset is observable to the depositors. But they cannot observe the size of the early withdrawals x or the risk behavior of their individual banks. Given z, depositors would demand a long term rate R^* such that their expected return for not withdrawing is no less than their outside option $\bar{r}$.

Secondary Market A secondary market for the long term assets is opened at date 1. Banks sell their long term assets for liquidity in the market at price p. There are unlimited number of potential buyers for the assets. Buyers are risk averse.

It will be shown later that the bank's choice of risk takes a corner solution, $\theta \in \{0, 1\}$. So the assets sold in the secondary market would yield either highly risky returns or riskless returns. There is asymmetric information between buyers and banks. Buyers cannot observe the risk associated with a specific asset. They know only that n^D fraction of assets have risky returns in the market. The risk averse buyers would demand a discount τ in price for holding the risky assets.

Therefore the market price p for the valuation of asset is given by

$$p^D(n^D, z) = \frac{z}{\bar{r}} \left(n^D(1-\tau) + (1-n^D) \right). \tag{5}$$

This section focuses on the basic version of the model. Later on, the model incorporates an ex ante choice of liquidity and capital respectively. The extensions of the model not only provides a more complete picture of the behavior of banks but it could generate more room for policy analysis as well.

[20] In the appendix, I relax this assumption. In stead of zero payoff when default, depositors can get a fraction of the gross return of the bank. Relaxing this assumption does not generate qualitatively different results.

3.1 Bank's Choice of Risk

The key decision the bank has to make is the choice of risk, which takes place at date 0. At date 0, the aggregate return on long term assets z is realized. Banks are risk neutral and only care about the expected payoff at date 2. Given z, the bank chooses risk θ to maximize its expected payoff:

$$\max_{\theta \in [0,1]} U_0(z;\theta) \tag{6}$$

The expected payoff of the bank

$$U_0(z;\theta) = E_x E_s \max\{y(z,x,s;\theta), 0\},$$

where y is the ex post payoff defined in Equation 4.

Assumption 1 $c < \frac{1}{3}$.

The assumption implies that the cost of taking risk is not high enough to discourage banks from taking risk regardless of z.

Lemma 1 *Under Assumption 1, for any value of z, $\theta = 0$ or 1.*

The proof is in the appendix. The intuition is the following. The bank faces a trade-off in the choice of risk. On one side, the bank incur a cost for taking risk. The cost is proportional to the asset returns. On the other side, by taking risk, the bank can benefit from risk shifting, in terms of both higher expected payoffs given x and higher probability of no default. The expected payoff first decreases then increases in θ.

The marginal payoff in risk is negative for θ sufficiently low. When a bank chooses low θ, the bank can receive payoffs in both positive and negative productivity shocks at date 2. Additional risk does not generate a high enough variation in the bank's survival probability between positive and negative productivity shocks. The marginal benefit from risk shifting is small. Meanwhile, the marginal cost of risk is $\kappa'(\theta)z$ per unit of asset, outweighing the marginal benefit.

The marginal payoff in risk becomes positive for θ high enough. As the variation in asset returns expands with θ, the bank generates payoffs only with positive productivity shock and always defaults in negative shocks. Additional risk yields return net of cost $(1-\kappa(\theta))z$ per unit of assets and costs $\kappa'(\theta)(1+\theta)z$. Under assumption 1, the marginal payoff in risk is positive for θ high enough.[21] The survival probability (in positive shocks) also rises when taking risk for $p < 1$. In this case, the expected payoff is increasing in θ.

Combining the two cases, the optimal choice of risk takes corner solutions, $\theta \in \{0, 1\}$.

[21] The linearity in $\kappa(\theta)$ is not essential. The result can be extended to $\kappa = c\theta^{\alpha}$, in which case the assumption on c is $c < \frac{1}{2\alpha+1}$.

PROPOSITION 1 (Optimal Choice of Risk) *Given the price of long term asset in the secondary market p and equilibrium long term deposit rate R^*, there exists a threshold $z^*(p, R^*)$ such that*

$$\theta(z, p, R^*) = \begin{cases} 1 & \text{if } z < z^*(p, R^*) \\ 0 & \text{if } z \geq z^*(p, R^*). \end{cases}$$

The threshold $z^*(p, R^*)$ is the following:

$$z^*(p, R^*) = \alpha(p) R^* \tag{7}$$

where $\alpha(p)$ is a function of secondary market price p and model parameters.

Proof: (In the Appendix).

The cost and benefit of risk taking differ according to different levels of z.

Figure 3 illustrates the intuition graphically. Without the cost of risk taking, risk taking is always preferred, as shown in the red dotted line. The bank obtains returns only in positive shocks when taking risk $\theta = 1$, so the benefit of risk taking comes from the higher survival probability (in positive shocks) and a lower expected repayment to late depositors.

The cost of risk taking is proportional to the returns on the long term assets. Higher mean return entails a higher cost of risk taking, as shown in the red solid line. After taking into account the cost, there exists a maximum mean return z^*, beyond which the bank prefer no risk in returns.

Because the bank's expected payoffs with and without risk are both homogenous of degree one in their arguments, an increase in R^* leads to an one-to-one increase in the threshold return z^*. The multiple $\alpha(p)$ is the ratio of return required to induce stable returns and the borrowing cost R^*. Intuitively, it is the liquidity premium the bank demands for exposing itself to liquidity shocks while not taking risk.

Proposition 2: *$\alpha(p)$ is increasing in p for $p < \tilde{p}$ and decreasing in p for $p \leq \tilde{p}$, where $\tilde{p} < 1$ and satisfies that*

$$(1 - c)x^H(\tilde{p}, 1, \alpha(\tilde{p}))^2 = x^L(\tilde{p}, 1, \alpha(\tilde{p}))^2, \tag{8}$$

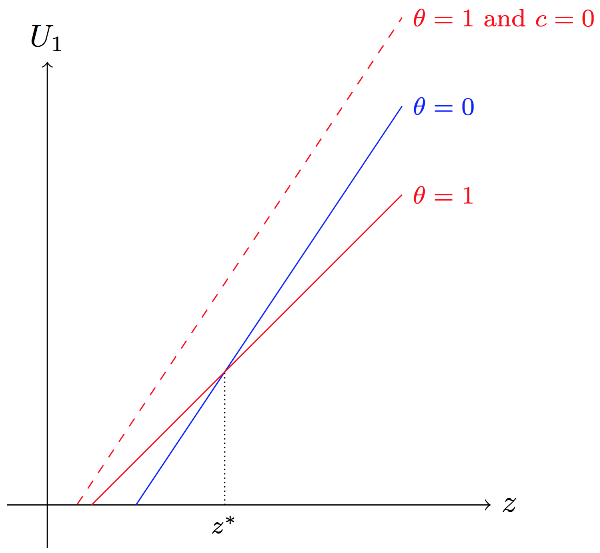

Figure 3: Threshold z^*

where x^H and x^L are the maximum sizes of early withdrawals that would not induce a default at date 2, when the bank take high risk (H) and no risk (L),

$$x^H(p, R^*, z) = \frac{z - \frac{R^*}{2(1-c)}}{\frac{z}{p} - \frac{R^*}{2(1-c)}} \tag{9}$$

and

$$x^L(p, R^*, z) = \frac{z - R^*}{\frac{z}{p} - R^*}. \tag{10}$$

Proof: (In the Appendix).

$\alpha(p)$ is non-monotonic in p. It first increases and then decreases in p. The intuition is the following.

For p sufficiently low, i.e., $p < \tilde{p}$, the secondary market is almost completely illiquid. Regardless of the choice of risk, the bank cannot sell long term assets for liquidity and is forced to default when hit by a liquidity shock. The bank can only generate positive payoffs when facing no early withdrawals. Without liquidity shocks, the bank has less incentive to take risk. Thus a lower p decreases the risk taking incentive.

For p high enough, the expected payoff given a liquidity shock increases with p. As p rises, the bank is further away from its default region, the risk shifting incentive diminishes. The direct effect becomes negative.

Figure 4 and 5 show the intuition graphically. A low market price p limits the bank's ability to withstand liquidity shocks, driving down its expected payoffs regardless of the choice of risk. The effect of p on the threshold z^* depends on how p changes the bank payoffs with and without risk. The responses of the expected payoffs depend on the survival probabilities and the cost of risk taking.

For p sufficiently low, the survival probability with risk taking (x_H) is significantly higher than with no risk (x_L). Thus the expected payoff with risk is more sensitive to changes in p. A decrease in p would depress the expected payoff with risk by more, leading to a lower risk shifting incentives.$z*$ falls as p falls. The direct effect is positive. See Figure 4.

As p rises, the survival probabilities both increase and the cost of risk taking starts to play a larger role by dampen the response of payoff with risk. The expected payoff without risk is more sensitive to changes in asset price. A decrease in secondary market price depresses the payoff without risk by more, leading to a higher risk shifting incentives. z^* rises as p falls. See Figure 5.

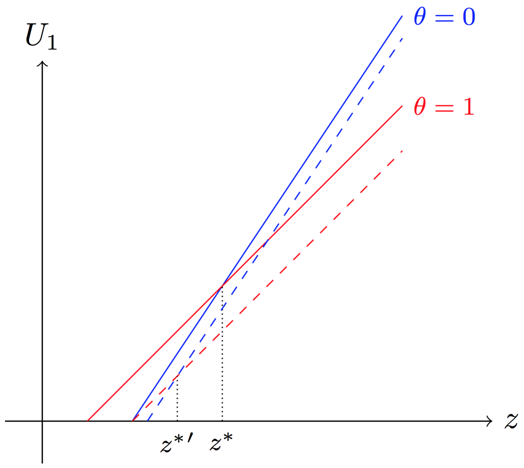

Figure 4: Change in z^* for a Decrease in p when p is low

3.2 The Determination of R^*

At date 1, observing the mean return on long term assets z and the secondary market price p, depositors form the probability of no default, taking into account the optimal choice of risk by banks and the expectation on x. The probability of no default from depositor's perspective γ^d is given by:

$$\gamma^d(z, p, R^*) = \begin{cases} Pr(y(z, x, s; 1) > 0) & \text{if } z > z^*(p, R^*) \\ Pr(y(z, x, s; 0) > 0) & o.w \end{cases}$$

For $p \leq 1$, banks can survive date 1 as long as the liquidity shock x is below the market value of bank asset, p. Banks can survive date 2 as long as their payoffs are high

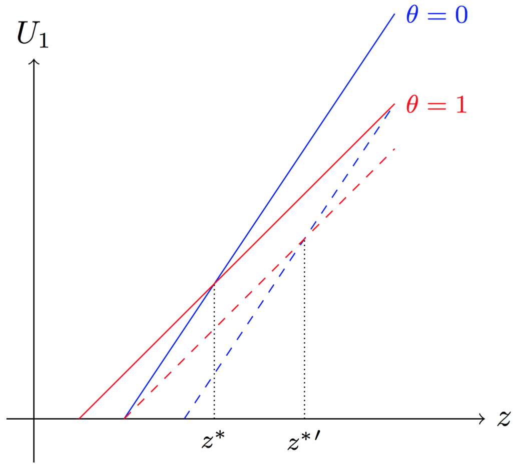

Figure 5: Change in z^* for a Decrease in p when p is high

enough to cover the repayment to late depositors.

$$\gamma^d(z,p,R^*) = \begin{cases} 0 & \text{if } z < \frac{R^*}{2(1-c)} \\ \frac{1}{2}\left(\lambda x_H(z,p,R^*) + (1-\lambda)\right) & \text{if } z \in [\frac{R^*}{2(1-c)}, z^*(p,R^*)] \\ \lambda x_L(z,p,R^*) + (1-\lambda) & o.w, \end{cases}$$

where x_H and x_L are defined in Equation 43 and 44 above.

For $p > 1$, the market value of bank asset is large enough to withstand liquidity shocks. Banks can always survive date 1. The bank will not default at date 2 as long as z is large enough.

$$\gamma^d(z,p,R^*) = \begin{cases} \frac{1}{2}\lambda\left(1 - x_H(z,p,R^*)\right) & \text{if } z < \frac{R^*}{2(1-c)} \\ \frac{1}{2} & \text{if } z \in [\frac{R^*}{2(1-c)}, z^*(p,R^*)] \\ 1 & o.w. \end{cases}.$$

Given the assumption that depositors will get nothing when bank defaults, the expected return for late depositors when they do not withdraw is $R^*\gamma^d(z,p,R^*)$. They demand deposit rate R^* such that their expected returns for not withdrawing at date 1 are no less than their outside option of investing in risk-less short term bonds with return $\bar{r}$, i.e.,

$$R^*\gamma^d(z,p,R^*) \geq \bar{r}. \tag{11}$$

In order to convince the depositors not to withdraw early, the bank has to offer a long term return R^* such that the participation constraint (Equation 11) above hold in equality.

Assumption 2 $(1-\lambda)(1-c)\underline{z} > \bar{r}$.

PROPOSITION 3 (The Existence of R^*) *Under Assumption 2, given the secondary market price p and mean return z, the equilibrium deposit rate $R^*(p,z)$ always exists for $p \geq 0$.*

Proof: (in the Appendix).

The proof is by the Intermediate Value Theorem. When banks offer a riskless rate, $R^* = \bar{r}$, because of the non-negative probability of default, the expected return for late depositors will always be less preferable than their outside option. When banks offer $R^* = 2(1-c)z$, the deposit rate is sufficiently large such that the banking sector always want to take risk in this case. By Assumption 2, the expected return of depositors when they do not withdraw at date 1 exceeds $\bar{r}$. To satisfy equation 11, there exists a unique equilibrium deposit rate $R^* \in [1, 2(1-c)z]$.

Lemma 2 *$R^*(p,z)$ is decreasing in p.*

Proof: (in the Appendix).

Secondary market price p indicates the difficulty in raising liquid assets by selling long term assets. It is linked to the notion of *"market liquidity"* as in Brunnermeier and Pedersen (2009). R^* is the borrowing cost the banks have to offer to their depositors. It reflects the bank's difficulty in financing its investment, or *" funding liquidity"* in Brunnermeier and Pedersen (2009).

The model suggests that in the presence of a liquidity shock, the secondary market illiquidity deteriorates the funding liquidity for banks by lifting up the credit risk. The mutual reinforcement between market liquidity and funding liquidity has been discussed in papers such as Diamond and Rajan (2005), Brunnermeier and Pedersen (2009), Krishnamurthy (2010) and Acharya, Gale, and Yorulmazer (2011), He and Xiong (2012) and He and Milbradt (2014).

3.3 Impact of p on Bank's Risk Taking

With long term deposit rate R^* endogenously determined, the effect of the secondary market price p on the threshold in risk taking decision z^* can be decomposed into two parts:

$$\frac{\partial z^*/z^*}{\partial p/p} = \underbrace{\frac{\partial \alpha}{\partial p}\frac{p}{\alpha}}_{\text{Direct Effect}} + \underbrace{\frac{\partial R^*}{\partial p}\frac{p}{R^*}}_{\text{Indirect Effect}} . \tag{12}$$

The secondary market price of the long term asset affects the choice of risk for banks through two effects. The first term summarizes the direct effect of p while keeping R^* fixed. Call this the *"direct effect"*. It reflects the change in the required liquidity premium to convince a bank not to take risk. As shown perviously, when p declines, the bank can withstand less liquidity shocks. It needs a higher return to compensate its exposure to the liquidity shock. That is , $\alpha(p)$ falls as p declines. Unless p is sufficiently low, in which case, the bank does not need a high liquidity premium to take risk because its payoff in liquidity shocks becomes negligible.

The second term summarizes the *"indirect effect"* of secondary market p on z^*. The channel is through the impact of p on the equilibrium long term deposit rate R^*.The indirect effect is always negative. A low p heightens the default probability of the bank and the depositors demand a higher return, R^*. The higher deposit rate drives down the bank's payoff with and without liquidity shock equally. Banks are unambiguously more likely to take high risk. z^* rises as a result.

The total effect of secondary market price on the optimal risk taking of the bank is the sum of the two effects. When p is high enough, both direct and indirect effects are negative. p encourages banks to take risk unambiguously. Immediately, we have the following result:

PROPOSITION 4 *For $p > 1$, both the direct and the indirect effects are negative, the total effect*

$$\frac{\partial z^*/z^*}{\partial p/p} < 0. \tag{13}$$

When price is high, the secondary market is liquid. Banks have no difficulty paying the early withdrawals. There is no liquidity-induced default at date 1. In this case, the choice of risk is not sensitive to changes in the secondary market price. First, the size of the direct effect is small, i.e.,

$$|\frac{\partial \alpha/\alpha}{\partial p/p}| = \frac{\lambda}{2p-\lambda} \leq 1.$$

Second, the size of the indirect effect is zero because R^* is either 1 or 2 for $p > 1$. R^* is insensitive to changes in p.

$$\frac{\partial R^*}{\partial p} = 0.$$

In sum, the total effect of p on z^* is negative and

$$\frac{\partial z^*/z^*}{\partial p/p} = \frac{\partial \alpha}{\partial p}\frac{p}{\alpha}. \tag{14}$$

As p rises further, it has less impact on the bank's expected payoff or choice of risk. The total effect becomes increasingly small.

For $p \leq 1$, the two effects have opposite signs for sufficiently low p. The total effect

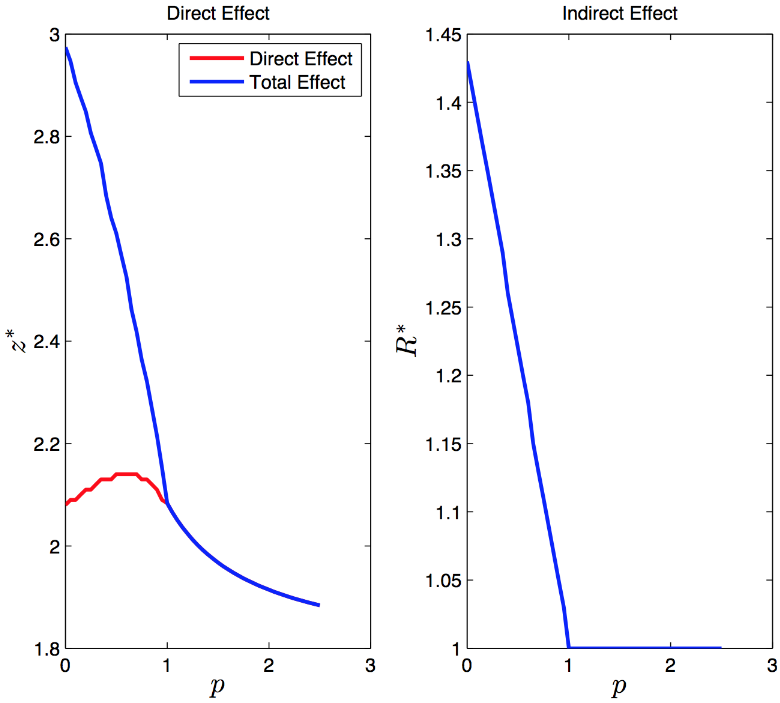

Figure 6: Threshold in the choice of risk, z^*

depends on which effect is dominant. Figure 6 illustrates separately the two effects of p on z^*. The left panel depicts the direct effect by fixing $R^* = 1$. The right panel illustrates the evolution of R^* given the mean return z. The numerical example suggests that the indirect effect is the driving force of the risk taking behaviors of banks. I formalize the discussion in the following:

Assumption 3 $(1-c)(1-\lambda)\underline{z} > \frac{1}{1-D(c)}$ where

$$D(c) = 2c\left((1-c)(\frac{1}{2c} - \frac{1}{2(1-c)})^2 - (\frac{1}{2c} - 1)^2\right).$$

PROPOSITION 5 *For $p \leq 1$, (i) under Assumption 3, the size of indirect effect always dominates that of the direct effect for $p < \tilde{p}$,*

$$|\frac{\partial R^*/R^*}{\partial p/p}| > \frac{\partial \alpha/\alpha}{\partial p/p} > 0;$$

(ii) both indirect and direct effects are negative for $p \in [\tilde{p}, 1]$; (iii) the total effect

$$\frac{\partial z^*(p)/z^*(p)}{\partial p/p} < 0. \tag{15}$$

for $p \leq 1$.

Proof is in the Appendix.

When p is sufficiently low, the two effects are of opposite signs. First, the size of the direct effect is small. The intuition is for sufficiently low p, the bank's payoff in liquidity shock is negligible while given R^*, its payoff with no liquidity shocks is not affected by p. As a result, the secondary market price p does not alter the choice of risk by much.

In comparison, the size of the indirect effect is relatively large. The equilibrium R^* is relatively more sensitive to changes in p, due to the mutual reinforcement of market liquidity and funding liquidity. To see this point clearly, the indirect effect can be further decomposed by differentiating Equation 11

$$\frac{\partial R^*/R^*}{\partial p/p} = -\underbrace{\frac{1}{1+\frac{\partial \gamma^d/\gamma^d}{\partial R^*/R^*}}}_{\text{multiplier}} \frac{\partial \gamma^d/\gamma^d}{\partial p/p}. \tag{16}$$

The second term in Equation 16 reflects the price elasticity on the survival probability of the bank. Under Assumption 3, it can be shown that the survival probability is more elastic in its response to p comparing with the requirement liquidity premium α,

$$\frac{\partial \gamma^d/\gamma^d}{\partial p/p} > |\frac{\partial \alpha/\alpha}{\partial p/p}|.^{22} \tag{17}$$

The first term in Equation 16 links the price elasticity of the survival probability with that of the deposit rate. Because[23]

$$0 < 1 + \frac{\partial \gamma^d/\gamma^d}{\partial R^*/R^*} \leq 1, \tag{18}$$

it acts as a multiplier that magnifies the response of the deposit rate to p after the initial response of the survival probability.

Combining the two inequalities,

$$|\frac{\partial R^*/R^*}{\partial p/p}| \geq \frac{\partial \gamma^d/\gamma^d}{\partial p/p} > |\frac{\partial \alpha/\alpha}{\partial p/p}|, \tag{19}$$

the dominance of the indirect effect can be established.

The dominance of the indirect effect is because market illiquidity erodes funding liq-

[22]The proof is in the appendix

[23]The Equation 18 holds because a) a high R^* dwindles the survival probability of the bank, $\frac{\partial \gamma_0}{\partial R^*} < 0$; and b) the depositors' expected return for not withdrawing is increasing when the deposit rate is at the equilibrium level, $\frac{\partial \gamma_0}{\partial R^*} > -1$.

uidity. As p falls, the bank entails a higher credit risk. Depositors demand a higher R^* as a compensation for taking higher risks. The first round of response of R^* is $\frac{\partial \gamma^d/\gamma^d}{\partial p/p}$. The higher borrowing cost pushes the bank closer to its default region, inducing more risk taking and driving up the credit risk further. R^* rises as a second round effect. The size of the second round response is $|\frac{\partial \gamma^d/\gamma^d}{\partial R^*/R^*}|\frac{\partial \gamma^d/\gamma^d}{\partial p/p}$.

The process goes on until the deposit rate R^* reaches the market equilibrium. The total impact of p on R^* :

$$|\frac{\partial R^*/R^*}{\partial p/p}| = \frac{\partial \gamma^d/\gamma^d}{\partial p/p}\big(1 + |\frac{\partial \gamma^d/\gamma^d}{\partial R^*/R^*}| + |\frac{\partial \gamma^d/\gamma^d}{\partial R^*/R^*}|^2 + ...\big). \tag{20}$$

Combining the two cases with different levels of the secondary market price, immediately we have the following result for the optimal risk-taking behavior for banks.

Lemma 3 *$z^*(p, z)$ is decreasing in p and z.*

A low secondary market price intensifies the risk shifting incentives and induces the bank to take more risk. A rise in the mean return on the long term asset reduces the credit risk and the deposit rate, leading to a lower threshold z^*.

3.4 The Aggregate Choice of Risk

Let $\hat{p}(z)$ satisfy that

$$z^*(\hat{p}, R^*(\hat{p}, z)) = z. \tag{21}$$

$\hat{p}(z)$ is the minimum secondary market price at which the bank starts to prefer no risk. The optimal choice of risk for individual bank can be rewritten as

$$\theta(p, z) = \begin{cases} 1 & \text{if } p < \hat{p}(z) \\ 0 & \text{if } p \geq \hat{p}(z). \end{cases}$$

The bank chooses risky returns when p is sufficiently low.

Because of the monotonicity of z^* in p and z, $\hat{p}(z)$ is decreasing in z. The bank has higher incentives to take risk for a low mean return z. Thus a high cutoff price is required in order to convince the bank not to take risk.

Denote n the fraction of banks that take risks in their long term investments. Given

p, the aggregate risk-taking behavior of the banking sector is

$$n(p,z) = \begin{cases} 1 & \text{if } p < \hat{p}(z) \\ [0,1] & \text{if } p = \hat{p}(z) \\ 0 & \text{if } p > \hat{p}(z). \end{cases} \tag{22}$$

The banking sector collectively take risk in long term assets when the secondary market price is lower than the cutoff price.

3.5 the Market Value for the Illiquid Asset

Buyers are risk averse and they demand a premium for holding asset with risky returns. There is asymmetric information between buyers and sells of assets. Buyers cannot identify the assets with risky returns. Let n^D denote the fraction of banks that take risks. With n^D probability, the buyer expects to obtain an asset with risky returns, which is valued at its expected return with a discount τ . So the valuation of the assets to the buyers is given by Equation 5.

More specifically, when no banks take risk, $p(0,z) = \frac{z}{\bar{r}}$. The price is the present value of the mean return. When all banks take risk, $p(1,z) = \frac{z}{\bar{r}}(1-\tau)$. Buyers discount the present value by τ.

The equilibrium is a fixed point problem where $p(n(p,z),z) = p$, where n is the aggregate risk-taking behavior of the banking sector.

3.6 Characterization of the Equilibrium

Definition of the equilibrium: *The rational expectation equilibrium is defined by* $(\theta^*, n^*, R^*, p^*)$ *such that*

a) θ^* *is the optimal choice of risk given the secondary market price* p^* *and equilibrium long term deposit rate* R^*.

b) n^* *is the proportion of banks that take risk given* p^* *and* R^*.

c) R^* *is the equilibrium long term deposit rate satisfying depositors participation constraint given* p^*.

d) p^* *is the valuation of long term asset by the buyers in the secondary market.*

The model could generate multiple equilibria. Figure 7 illustrates an example of that. The equilibrium outcome depends on the expectation on the secondary market price.

In expectation of a high price in the secondary market, banks have less concerns of liquidation and fire sales. With high expected payoffs, banks have less incentives to take risk in long term investments. The volatility of asset returns is low and the price in the

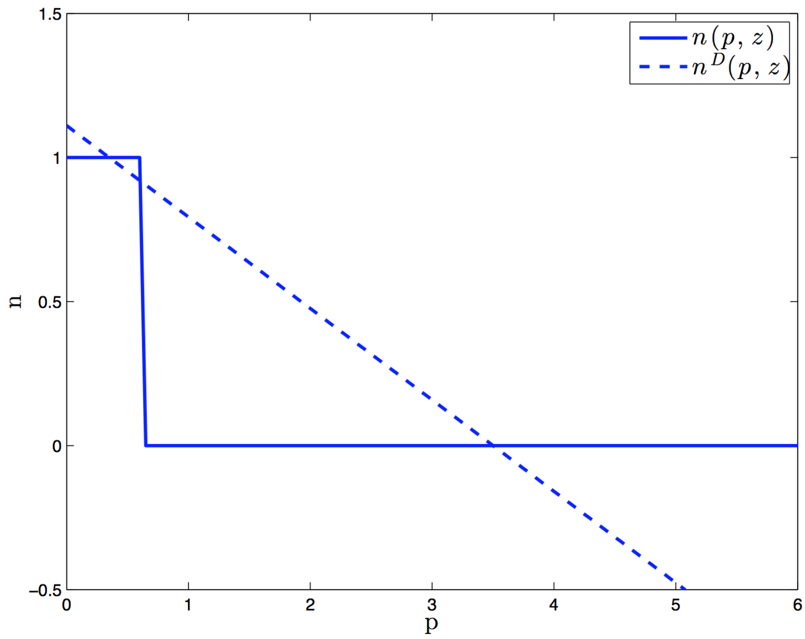

Figure 7: Market Equilibrium

secondary market is consistent with the initial expectation. In this case, the economy reaches the *low volatility equilibrium*.

On the contrary, when a high fire sale discount in the secondary market price is expected, banks have difficulty in paying for their early withdrawals. Close to the default region, banks have more incentives to take risk to boost their payoffs. Consequently, the credit risk rises, inducing more banks to take risk. In equilibrium, the whole banking sector takes high risk for their long term investments. With the returns more volatile, the asset price in the secondary market is indeed low. It is the *high volatility equilibrium*.

Both equilibria are locally stable. Any perturbation to the equilibrium price will not persist. The equilibrium at $p = \hat{p}$ is unstable.

Multiple equilibria exist in this model because of the strategic complementarity generated by the risk-taking behavior of banks. When banks choose risk for their portfolios, they do not take into account the downward pressure they exert on the asset price in the secondary market. A low price in the secondary market in turn depresses payoffs of other banks. As a response to lower payoffs, other banks are encouraged to take risk. Negative externalities of the risk-taking behavior of banks give rise to a systemic incentive to take risk. As a result, the long term assets in the economy may end up with highly volatile returns.

PROPOSITION 5 (Existence of Multiple Equilibria) *Given the mean aggregate return z, (i) The low volatility equilibrium exists when $\hat{p}(z) \leq p^D(0,z)$; (ii) The high volatility equilibrium exists when $\hat{p}(z) \geq p^D(1,z)$; (iii) Multiple equilibria exists when (i) and (ii) hold simultaneously.*

The low volatility equilibrium exists when the buyers value the assets with risk-less returns high enough to convince the banking sector not to take risk. Meanwhile, the high volatility equilibrium exists when the buyers value the risky assets low enough to encourage collective risk taking of the whole banking sector.

When the social planner takes the payoffs of both depositors and banks into consideration, in the presence of multiple equilibria, the low volatility equilibrium is more efficient than the high volatility equilibrium. The reason is that while the depositors expected payoff are equal in both equilibria[24], the bank's payoff is always higher in the low volatility equilibrium.[25]

3.7 Comparative Statics

This subsection studies how the aggregate productivity z, the probability of a liquidity shock λ and the cost of taking risk c affect equilibrium outcomes.

3.7.1 Aggregate Productivity z

Aggregate productivity boosts long term payoffs of banks and weakens their incentive to take risk. In the presence of a high z, a large fire sale discount in the secondary market price is needed to convince the banking sector to take risk. As shown previously, $\hat{p}(z)$ is decreasing in z.

Let z_G denote the threshold productivity which satisfies that

$$\hat{p}(z) = p^D(0,z). \tag{23}$$

Because $\hat{p}(z)$ is decreasing in z, the condition for the existence of the good equilibrium (with low volatility) is equivalent to $z > z_G$.

Similarly, let z_B denote the threshold productivity such that

$$\hat{p}(z) = p^D(1,z). \tag{24}$$

[24] This is followed by the depositors participation constraint.

[25] It can be shown that

$$U_0(z;1)|_{p=p^D(1,z)} \leq U_0(z;1)|_{p=\hat{p}(z)} = U_0(z;0)|_{p=\hat{p}(z)} \leq U_0(z;0)|_{p=p^D(0,z)}.$$

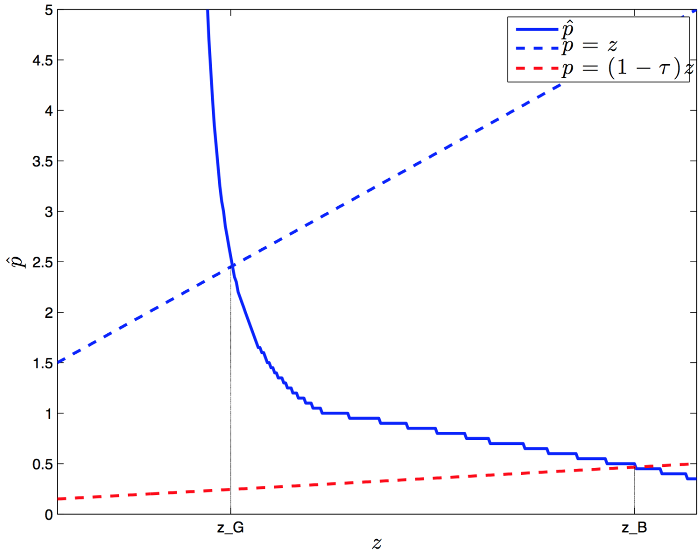

Figure 8: Threshold $\hat{p}$ as a function of z

The condition for the bad equilibrium (with high volatility) is equivalent to $z < z_B$.

Note that $z_G < z_B$. Multiple equilibria exists when $z \in [z_G, z_B]$. In this range, expectations can be self-fulfilling. See Figure 8.

The effect of aggregate productivity on risk taking decision by banks works through two channels. Given R^*, a low mean return z encourages banks to take advantage of the limited liability to boost payoffs. More banks take risk. Expecting this, depositors demand higher rate R^* to compensate for the default risk, which intensifies the risk-shifting incentive further. $\hat{p}$ rises as z falls. A wider range of secondary market price is admissible for the systemic risk-taking behavior of the banking sector. The high volatility equilibrium becomes more likely. When the mean return z is sufficiently low, the economy can potentially be trapped in the inefficient equilibrium with severe fire sale discounts and high volatility in asset returns. The Figure 8 illustrates the domain of z for multiple equilibria in a numerical example with given c and λ.

3.7.2 Changes in c

c denotes the cost of taking high risk, i.e., $c = \kappa(1)$. An increase in c means banks incur higher costs when taking risky investments. The threshold $\hat{p}$ is decreasing in c. The intuition is the following. For given R^*, the bank's payoff without taking risk is not affected. Meanwhile, the payoff with risk-taking falls. As it becomes costly to take risky investments, the bank is less willing to do so. In equilibrium, depositors expect less defaults and demand a lower deposit rate. The incentive to take risk falls even further.

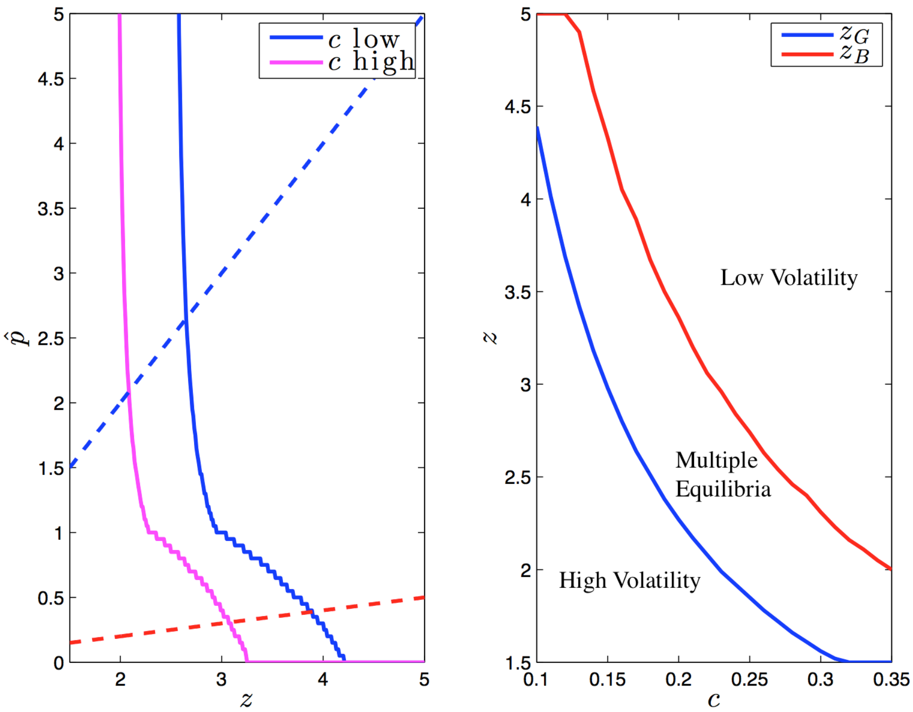

Figure 9: Threshold $\hat{p}$ with Changes in c

As c rises, a lower secondary market price is needed to convince the whole banking sector to take risk. The left panel in Figure 9 depicts the cutoff price $\hat{p}$ with a change in c. As c rises, both boundary conditions for the good and bad equilibria (z_G and z_B) decreases. With a given λ, the right panel illustrates the two boundary conditions z_G and z_B in c. In between is the region for multiple equilibria. Because c reduces risk-taking incentive, as c rises, the good equilibrium becomes more likely and the bad equilibrium becomes less likely.

3.7.3 Changes in λ

λ is the probability that a liquidity shock hits the banking sector at date 1. The effect of λ on the expected payoffs of banks is twofold. On the one hand, a high λ depresses the bank payoffs because with the rising needs for early withdrawals, the banks are more likely to sell assets for liquidity. On the other hand, a high λ improves the bank payoffs because as more depositors are expected to withdraw early, less repayment is needed in the long term.

The total effect of λ on the risk decisions of banks depends on the secondary market price. For $p > 1$, the bank can always withstand liquidity shocks at date 1. The second

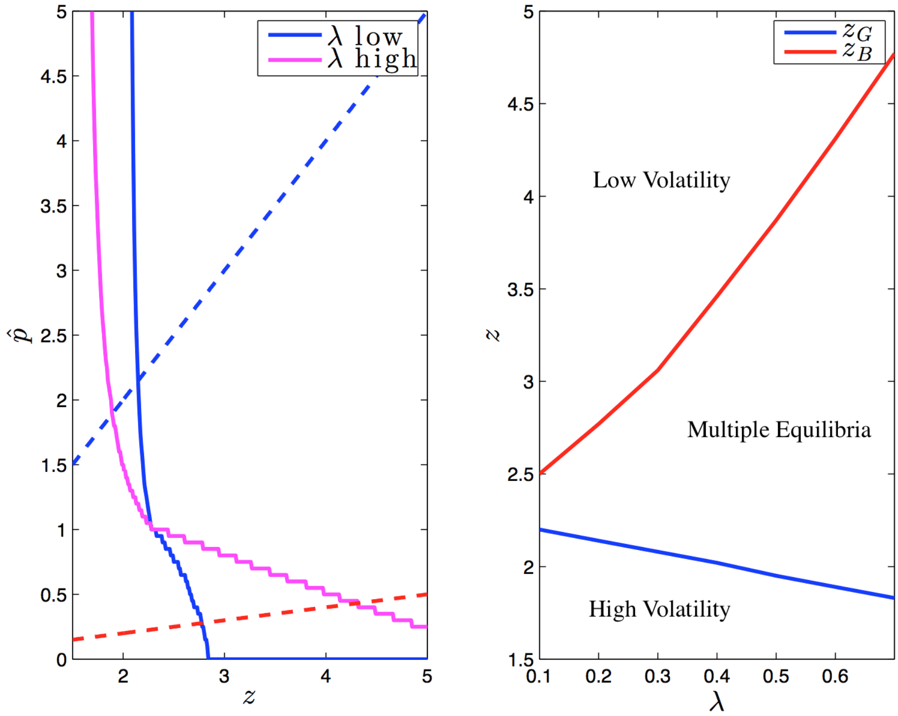

Figure 10: Threshold $\hat{p}$ with Changes in λ

effect dominates. The expected payoffs increase in λ because banks expect lower total repayments to the late depositors at date 2. Therefore a high λ discourages risk-taking behavior. As banks expect a higher probability of liquidity shocks, they would not take risk unless the secondary market price is low enough, i.e. $\hat{p}$ falls.

For $p \leq 1$, with an less liquid secondary market, banks become more concerned over the forced fire sales. The first effect dominates. A high λ lowers bank payoffs and encourages risk taking. Meanwhile, since defaults become more likely, depositors demand a higher R^*. Banks are pushed even closer to their default region, intensifying their incentives to take risk. $\hat{p}$ increases with λ. A wider range of secondary market prices is admissible for the existence of the high volatility equilibrium.

The left panel in Figure 10 illustrates the cutoff price $\hat{p}$ in z. λ affects $\hat{p}$ differently depending on the secondary market price. As discussed above, an increase in λ reduces risk-taking when p is high. $\hat{p}$ decreases, rendering the good equilibrium more likely. On the contrary, it encourages risk-taking when p is low. $\hat{p}$ increases, raising the likelihood of the bad equilibrium. The right panel illustrates the boundary conditions for the good and bad equilibria. An increase in the liquidity shock probability λ raises the likelihood for both equilibria, leading to an expansion of the region for multiple equilibria.

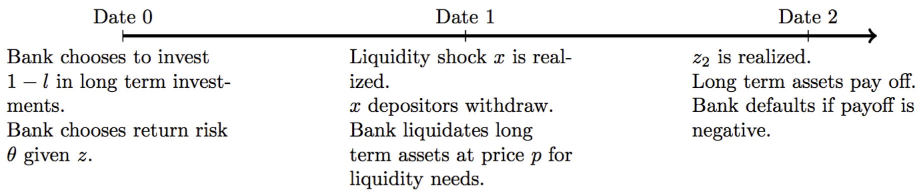

Figure 11: Time-line with Ex Ante Choice of Liquidity

4 The Model with Ex Ante Liquidity

This section studies an extended version of the model by incorporating an ex ante choice of liquid asset holding. That means, at date 0, the bank can invest in a safe and perfectly liquid asset, which generates return 1 from date 0 to date 1. Now, banks are making two decisions at date 0: given mean return z, banks choose both the quantity and the risk of their long term investments.[26]

With the additional choice of liquidity holding, the time line in the extended model is illustrated in Figure 11.

When the bank is subject to liquidity shocks, it is natural to discuss bank's problem where the bank has the option to hold liquid assets ex ante as a precautionary buffer. Incorporating the ex ante choice of liquidity also provides more room for the discussion of some maco-prudential policies.

The effect of liquidity holding on the choice of risk is twofold. On one hand, the holding of liquidity prevents long term assets from liquidating so it boosts long term payoffs of banks, which lower their risk-shifting incentives. On the other hand, when the bank hoards liquidity in stead of engaging in long term investments, the bank payoff is negatively affected. Consequently, the bank tends to take more risk. Which effect dominates will depend on the relative strengths of the cost and the benefit of holding liquidity. The model suggests that the total effect of liquid asset holding on the risk taking by banks is non-monotonic. It will depend on the secondary market price.

[26] The choice of liquid asset holding can happen before the mean return z. In this case, assuming that z is drawn from a distribution $F(z)$ on $[\underline{z}, \bar{z}]$. This setting means that the bank commits certain funds to some long term projects and then chooses the risk associated with the project given the realization of z. The timing here does not alter the results significantly.

4.1 Bank's Problem

Now banks have two decisions to make. Given an aggregate mean return z, banks decide how much liquidity to hold and the riskiness on their long term assets simultaneously:

$$\max_{l,\theta\in[0,1]} U_0(l, z; \theta). \tag{25}$$

U_0 is the expected payoff of the bank,

$$U_0(l, z; \theta) = E_x E_s \max\{y(l, z, x, s; \theta), 0\}, \tag{26}$$

where the ex post payoff

$$y(l, z, x, s; \theta) = \begin{cases} (1-l)(1+s\theta)(1-\kappa(\theta))z + (l-x)\bar{r} - (1-x)R^* & \text{if } x < l \\ (1-l-\frac{x-l}{p})(1+s\theta)(1-\kappa(\theta))z - (1-x)R^* & o.w. \end{cases}$$

In order to fulfill the liquidity needs, the bank liquidates its long term asset after it exhausts its liquidity buffer l.

4.1.1 Choice of Risk

I first solve for the choice of risk by banks for given liquidity holding l and then solve for the optimal liquidity holding. Similar to the previous analysis, the optimal choice of risk would take corner solutions, i.e., $\theta \in \{0, 1\}$.

PROPOSITION 6 (Choice of Risk) *Given mean return z and the choice of liquidity l, bank chooses riskiness of its long term assets θ according to*

$$\theta(l, z, p, R^*) = \begin{cases} 1 & \text{if } z < z^*(l, p, R^*) \\ 0 & \text{if } z \geq z^*(l, p, R^*) \end{cases} \tag{27}$$

where $z^*(l, p, R^*)$ is the threshold mean return that equates the expected payoffs with and without risk,

$$U_0(l, z; 1) = U_0(l, z; 0). \tag{28}$$

Because the expression for $z^*(l, p, R^*)$ is complicated to compute, there is no closed-form solution for z^*. However, we can discuss the evolution of z^* in its arguments.

Lemma 4 *$z^*(l, p, R^*)$ is increasing in l for small enough λ.*

The proof is in the Appendix.

The lemma suggests that the choice of risk and quantity for long term investments are substitutes. When the bank holds more liquid assets, less is invested in long term projects. In order to boost payoff, the bank takes more risk.

In general the total effect of individual liquidity holding l on bank payoffs and z^* will depend on the secondary market price. For secondary market price $p > 1$, $z^*(l, p, R^*)$ is increasing in l unambiguously. Holding liquidity depresses payoffs because the cost from the forfeited returns on long term investments exceeds the benefit of holding a stronger liquidity buffer. With the cost of taking risk c, the expected return on long term investment per unit of fund is c fraction lower for banks that take risk. So the expected payoff without risk falls by more for a given incremental increase in liquidity holding. As a result, liquidity holding induces risk taking behavior.

For secondary market price $p \leq 1$, the monotonicity of z^* in l may not hold in general. Holding one unit of additional liquidity saves more than one unit ($\frac{1}{p} \geq 1$) of assets from liquidation. The gain of holding liquidity can potentially outweigh the cost. The expected payoffs given positive liquidity shocks with and without risk can potentially be increasing in liquidity holding, in which case the payoff without risk taking improves by more for a given incremental increase in l. Banks will be discouraged to take more risk unless λ is small enough. For sufficiently small λ, banks put more weight on the expected payoffs conditional on no liquidity shocks, where the concern for not able to withstand liquidity shocks is less significant comparing with the loss from forfeited return. Similar to the case where $p > 1$, holding liquidity adversely affects the expected payoffs and the expected payoffs without risk taking responds by more. Again, the increasing property of z^* in l holds. Liquidity holding facilitates risk taking behavior.

4.1.2 Choice of Liquidity

Given the realization of z and its optimal choice of risk in Equation 27, the bank chooses optimal liquidity l to optimize its expected payoff,

$$\max_{l \in [0,1]} U_0(l, z; \theta^*). \tag{29}$$

Proposition 7 (Optimal Liquidity Holding) *For λ sufficiently small, $l^* = 0$ for all z and p.*

The proof is in the appendix. Under some conditions for λ, the bank has no incentive to hold liquid assets ex ante regardless of the secondary market liquidity.

The intuition is the following. At date 0, banks weigh the tradeoff in holding liquid assets. Given the realization of z, additional liquid asset holding is beneficial as it prevents

$\frac{1}{p}$ long term assets from liquidation. On the other hand, liquidity holding incur a cost since the bank has to give up returns from long term assets as long as it does not default. In a liquid secondary market with $p > 1$, the cost of liquidity holding always exceeds the benefit for all realizations of z. It is optimal not to hold any liquidity ex ante.

In an illiquid secondary market with $p \leq 1$, holding additional liquidity saves more than one unit ($\frac{1}{p} \geq 1$) of assets from liquidation. The gain from holding a stronger liquidity buffer can potentially outweigh the cost. Moreover, as p falls, it becomes increasingly difficult to sell long term assets for liquidity. Banks benefit more from holding liquid assets ex ante. Banks prefer holding non-zero liquidity when p is sufficiently low.

For λ sufficiently small, the ex ante illiquidity risk is small. That is, the bank is very likely to survive date 1. In this case, the bank has no incentive to hold liquid assets ex ante even for a low p. Therefore, the extended version of the problem coincides with the basic model.

4.2 The Determination of R^*

Same as in the basic model, depositors observe the realized mean return z and the asset price in the secondary market p. They cannot observe the decisions (on liquid assets holding or risk taking) made by their individual banks. Nor can they observe the size of the liquidity shock x. They form the probability of no default, taking into account the optimal choices of risk and liquidity holding by banks. The probability of no default from depositor's perspective γ^d:

$$\gamma^d(l^*, z, p, R^*) = \begin{cases} Pr(y(l^*, z, x, s; 1) > 0) & \text{if } z < z^*(l^*, p, R^*) \\ Pr(y(l^*, z, x, s; 1) > 0) & \text{if } z \geq z^*(l^*, p, R^*) \end{cases} \tag{30}$$

The equilibrium long term deposit rate R^* the bank offers has to satisfy the depositors' participation constraint:

$$R^* \gamma^d(l^*, z, p, R^*) = \bar{r}. \tag{31}$$

Given λ sufficiently small, $l^* = 0$. The equilibrium R^* is identical to the solution in the basic model. Incorporating the equilibrium R^* into z^*, we can rewrite z^* as a function of secondary market price p and aggregate mean return z.

Lemma 5 *For λ sufficiently small, z^* is decreasing in p.*
The evolution of z^* is the same as in the basic model. The lemma holds immediately by Lemma 3.[27] Low price in the secondary market adversely affects the bank's payoffs and

[27]The result still holds when the bank has a mild incentive to hold liquid assets ex ante. When λ is large and the incentive to hold liquid assets is strong, the negative indirect effect on z^* through the

encourages risk-taking through a combination of direct and indirect effects.

4.3 The Aggregate Risk Taking of the Banking Sector

Due to the monotonicity of z^* in p, similar to the analysis in the basic model, there exists a threshold secondary market price $\hat{p}(z)$ and it satisfies that

$$z = z^*(0, p, R^*(p, z)). \tag{32}$$

The banking sector take high risk in long term investments when p is sufficiently low, i.e., $p < \hat{p}(z)$.

Given secondary market price p and the mean return z, the fraction of banks that take risk in equilibrium is

$$n(p, z) = \begin{cases} 1 & \text{if } p < \hat{p}(z) \\ [0, 1] & \text{if } p = \hat{p}(z) \\ 0 & \text{if } p > \hat{p}(z). \end{cases} \tag{33}$$

4.4 Characterization of the Equilibrium

The buyers' valuation of the long term asset is in Equation 5.

Definition of the equilibrium: *The rational expectation equilibrium is defined by $(l^*, \theta^*, n^*, R^*, p^*)$ such that*

a) l^* and θ^* are the optimal choices of liquidity and risk in asset returns chosen by the banks given p^* and R^*.

b) R^* is the equilibrium long term deposit rate satisfying depositors participation constraint given p^* and optimal choices by the banks, l^* and θ^*.

c) n^* is the proportion of banks that take risk given p^* .

d) p^* is the valuation of long term asset by the buyers in the secondary market.

Similar to the basic model, the condition for the existence of multiple equilibria is the following:

PROPOSITION 8 (Existence of Multiple Equilibria) *Multiple equilibria exist when $\hat{p}(z) \in (p^D(1, z), p^D(0, z))$, where $\hat{p}(z)$ is defined in Equation 32.*

deposit rate may be partially offset by the liquid asset holding. The reason is that the high deposit rate lowers the bank's survival probability, leading to a decrease in the benefit of holding cash ex ante. A low liquidity buffer ex ante lessens the risk shifting incentives by Lemma 4.

In the low volatility equilibrium, the secondary market is liquid with $p > 1$. Expecting low volatility in asset returns, banks will not hoard liquid asset ex ante because they expect that it will be easy to satisfy the liquidity need of its depositors. The equilibrium behaves exactly the same as the one in the basic model. With all its funds tying to the long term investments, the bank has less incentive to take risk because the cost of risk taking exceeds the benefit. So the volatility of asset returns is low.

High volatility equilibrium exists when the buyers valuation for the risky assets is low enough to induce the banking sector to take risk. Risky asset returns is associated with lower price in the secondary market. With low λ, the likelihood of a positive liquidity shock is low, and banks would not exploit the extra option of storing values. In response to a low price, banks choose risky projects. As in the basic model, the expectation of a low price in the secondary market can be self-fulfilling.

4.5 Liquidity Requirement

Because the equilibrium with the self-fulfilling crisis generates welfare loss, this section analyzes how some standard macro-prudential policies affect the financial market efficiency in the context of the model. Specifically, consider a liquidity requirement imposed on the whole banking sector. Banks are required to hold at least κ units of liquid assets for each unit of deposits. So there is an additional constraint in the bank's problem in Equation 25,

$$l \geq \kappa.$$

For sufficiently low λ, banks have no incentives to hold liquid assets ex ante. The liquidity requirement always has binding power.

Lemma 6 *For κ small enough, the effect of liquidity regulation on risk-taking z^*:*
(i) For $p > 1$,

$$\frac{\partial z^*(\kappa, p, R^*)}{\partial \kappa} > 0. \tag{34}$$

(ii) For $p \leq 1$,

$$\frac{\partial z^*(\kappa, p, R^*)}{\partial \kappa} < 0. \tag{35}$$

As the liquidity regulation applies to all banks in the economy, a change in κ affects z^* through two effects. First, it raises the liquidity holding of individual banks given that the liquidity regulation has binding power. z^* rises as banks are holding more liquid assets according to Lemma 4. In addition to the impact on individual banks, change in κ lowers the risk taking incentive through aggregate effects. As the aggregate liquidity holding of banking sector rises, depositors expect a fall in the credit risk as well as in the long term deposit rate. Combining the two effects, in the presence of the liquidity requirement, the threshold in risk-taking behavior z^* falls for $p \leq 1$ and rises for $p > 1$.

When $p > 1$, all banks have enough liquidity to cover the early withdrawals and they can always survive date 1. Banks only default when they take risky investment and experience the negative productivity shock at date 2, which takes place with 1/2 probability. Additional liquidity holding will affect neither the credit risk nor the deposit rate. The effect of liquidity regulation on z^* is identical to the effect of individual liquidity holding as discussed in the previous section. When secondary market price is expected to be high, a liquidity requirement will be counterproductive by encouraging the banking sector to take more risk.

When $p \leq 1$, the liquidity requirement $\kappa > 0$ can discourage risk taking by banks. The intuition in the following. In addition to positive effect of liquidity holding on risk-taking at individual levels as discussed previously, the increase in the aggregate liquidity holding affects the equilibrium deposit rate R^*. With $p < 1$, early withdrawals may potentially exceed the market value of bank asset ($p(1-l) + l < 1$), in which case the bank will be forced to default at date 1 due to a liquidity shortage. Holding more liquid assets strengthens bank's ability to withstand liquidity shocks and reduces the illiquidity risk and hence the credit risk of the bank. Therefore, bank payoffs improve as depositors demand a lower deposit rate. In this case, with improvement in bank payoffs, a liquidity requirement can rein in risk taking by banks.

4.5.1 Effect of Liquidity Regulation on Multiple Equilibria

Given liquidity regulation κ, the aggregate choice of risk of the banking sector becomes

$$n(p,z) = \begin{cases} 1 & \text{if } p < \hat{p}(z;\kappa) \\ [0,1] & \text{if } p = \hat{p}(z;\kappa) \\ 0 & \text{if } p > \hat{p}(z;\kappa) \end{cases} \tag{36}$$

where the cutoff price $\hat{p}(z;\kappa)$ is defined as

$$z = z^*(\kappa, p, R^*). \tag{37}$$

Proposition 8 (The Effect of κ on Aggregate Risk Taking) *For κ small enough, the effect of liquidity requirement κ on the aggregate risk taking:*

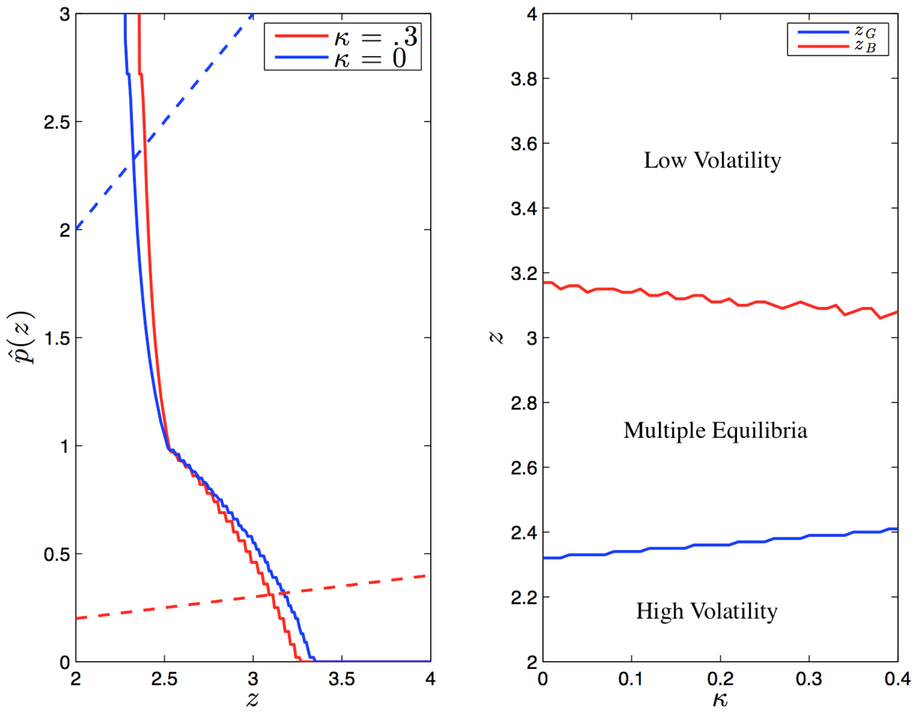

Figure 12: The Effect of κ on risk taking and on equilibrium

(i) For $p > 1$,

$$\frac{\partial \hat{p}(z;\kappa)}{\partial \kappa} > 0. \tag{38}$$

(ii) For $p \leq 1$,

$$\frac{\partial \hat{p}(z;\kappa)}{\partial \kappa} < 0. \tag{39}$$

The proposition can be shown immediately by Lemma 6. The model suggests that the effect of a liquidity requirement is ambiguous in improving financial stability, as shown in Figure 12.

When the secondary market price is low, banks are holding liquid assets at a level which is lower than the social optimal level. The reason is that banks do not internalize the effect of their liquidity holdings on the equilibrium credit risk and deposit rate. Liquidity requirement reduces the risk-shifting incentives by reducing the credit risk and hence the borrowing cost of banks and improving bank payoffs. With less willingness to take risk, a lower secondary market price is required to convince banks to take risk. The cutoff price shifts down for $p \leq 1$, as shown in the left panel of Figure 12. The high volatility equilibrium becomes less likely to exist. In another word, less z's are admissible for the existence of the high volatility equilibrium, as shown in the right panel of Figure 12. Liquidity requirement is welfare-improving by enhancing financial stability for $p \leq 1$.

However, when the secondary market price is expected to be high, the liquidity requirement is potentially counterproductive in stabilizing the financial system. The cost of forfeited long term returns outweighs the benefit from a stronger liquidity buffer. Liquidity requirement restrains banks from making long term investments and dampen their long term payoffs. As a result, banks are incentivized to rely on risky returns to boost payoffs. The excessive risk taking reduces the occurrence of the low volatility equilibrium. An increase in κ would intensify the risk-shifting incentives further and render the low volatility equilibrium less likely to exist. In Figure 12, the cutoff price shifts up for $p > 1$ and less z's are admissible for the existence of the low volatility equilibrium.

In sum, the liquidity requirement reduces the occurrence of both the good and bad equilibria. The model suggests that the liquidity requirement imposes a tradeoff in the policy making process: improving financial stability in bad times (by reducing the high volatility equilibrium) vs encouraging excess risk taking in good times (by reducing the high volatility equilibrium).

4.5.2 Discussions on the Counter-cyclical Liquidity Requirement

The recent financial crisis sheds light on the pro-cyclicality of behaviors in the financial market. There has been a growing consensus on the implementation of the counter-cyclical regulations in promoting the resilience of the financial system.

The model provides theoretical arguments in favor of the implementation of counter-cyclical liquidity requirement, in which the liquidity requirement is raised during economic upturns and lowered during economic downturns. As discussed above, a constant liquidity requirement imposes a tradeoff between improving financial stability (by reducing the high volatility equilibrium) vs. encouraging excess risk taking (by reducing the low volatility equilibrium). The counter-cyclical liquidity requirement is shown to promote financial stability by improving the tradeoff. The intuition is the following.

The relative strength of the two sides of the tradeoff of a liquidity requirement differs according to different levels of aggregate productivity. When the aggregate productivity is high, a high liquidity requirement guarantees that banks hold enough liquid assets as precautionary buffers for the upcoming liquidity shocks. The requirement could effectively rein in excessive risk taking when fire sales are expected. Meanwhile, it would not encourage risk taking significantly because during economic booms, long term projects generate high returns and the bank total payoffs are less affected by holding more liquid assets.

On the contrary, when the aggregate productivity is low, a low liquidity requirement would reduce the incentive distortion and discouraging excessive risk taking when the secondary market price is high. Meanwhile, the requirement may not further intensify the risk taking incentives because during economic downturns, bank payoffs are already low.

The model suggests that the liquidity requirement should be made counter-cyclical. In this way, we can utilize the beneficial effect of a liquidity requirement while minimize its counter-productive effect on the financial market in order to promote financial stability.

In fact, the optimal liquidity requirement implied by the model is given by solving the constrained first best problem that maximizes the aggregate expected payoff of banks,

$$\max_{\kappa} U_0(\kappa, z; \theta^*). \tag{40}$$

subject to the depositors participation constraint,

$$R^* \gamma^d(\kappa, z, p, R^*) = \bar{r} \tag{41}$$

and the optimal risk taking of individual banks,

$$\theta^*(\kappa, z, p, R^*) = \begin{cases} 1 & \text{if } z < z^*(\kappa, p, R^*) \\ 0 & \text{if } z \geq z^*(\kappa, p, R^*). \end{cases} \tag{42}$$

As illustrated in Figure 13, the resulting cutoff price for the aggregate risk taking by banks, $\hat{p}(z; \kappa^*(z))$ is everywhere below the cutoff price with a constraint liquidity requirement. The countercyclical liquidity requirement improves financial stability by reducing risk taking incentives for all z.

4.5.3 Discussions on the Interest Rate Policy

In recent years, negative interest rates have been widely discussed and deployed in countries including the euro zone, Switzerland, and Japan. The implementation of a negative interest rate means that the commercial banks are charged for holding excess reserves with the central bank. Policy makers believe the negative rates can help boost the economy in several ways. First, the sub-zero rate encourages commercial banks to make more loans to avoid charges on cash hoarding. Second, it could help banks raising fund by lowering the borrowing cost and increasing the relative appeal of equities.

However, the model suggests that such policy potentially generates unintended consequences. The bank may not be able to pass on the lower rate to its depositors and

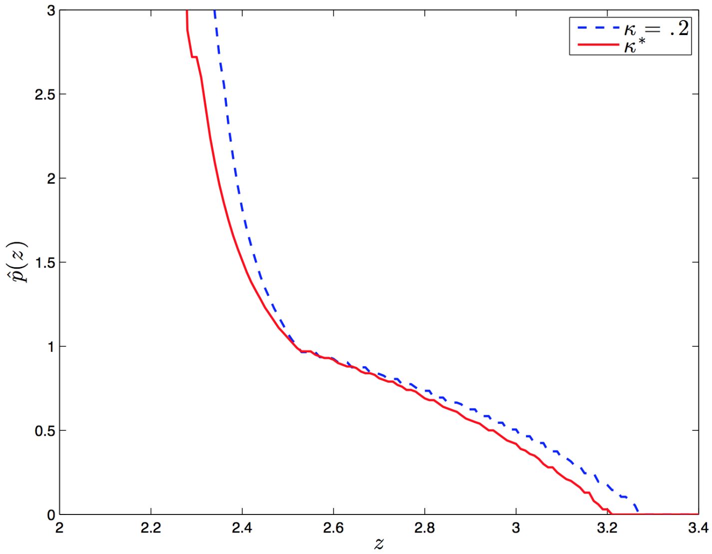

Figure 13: The Effect of Optimal Liquidity Requirement κ^* on risk taking

other short term creditors, because their returns from outside options are limited by the zero-interest cash. Due to the downward rigidity in the returns of creditors from outside options, the bank profits are squeezed. With lower profit margins, the bank would prefer investing in riskier projects for the benefit of risk-shifting to boost its payoff. As a result, the credit risk of banks rises. In stead of a lower borrowing cost intended by the sub-zero rate policy, the banking sector could potentially face a higher borrowing cost, leading to further disruptions in the banking sector. The incentive to "search for yield" [28] imposes a challenge for the stability of the financial system.

Empirical evidence confirms the existence of the "search for yield" effect. In periods of low (but positive) interest rates, banks indeed lowered their lending standard and shifted their credit to less qualified creditors.[29] Recently, the latest ECB Survey on the Access to Finance of Enterprises in the Euro Area (SAFE) shows that bank loans for SMEs become more available after the introduction of the negative interest rate.[30]

The paper suggests that the negative interest rate policy can generate some unintended consequences that the policy makers need to pay attention to. Whether or not the negative rate is effective in boosting the real economy needs a more comprehensive examine and it is beyond the scope of this paper. The paper wants to bring attention the potential that incentives of the financial sector play a role in affecting the effectiveness of the interest

[28]See Rajan (2006)

[29]See, for example, Ioannidou, Ongena and Peydro (2009), Maddaloni and Peydro (2011), Jimenez et al (2014) and Dell'Ariccia, Laeven and G. Suarez(forthcoming).

[30]See March 2016 SAFE surveys

rate policy.

Moreover, the negative rate may not be enough to cure economic recessions. Neither is it the only policy tool on the menu for central banks. To mitigate the adverse effects of the negative rate policy, other unconventional monetary policy tools such as quantitative easing and forward guidance should be considered.

5 Conclusion

The paper points out a novel mechanism through which a deterioration of financial health in the banking sector would affect the financial stability. The channel is through risk taking of banks. In the model, the risk taking behavior of banks gives rise to strategic complementarity. When in need of liquid assets to pay for the early withdrawals, banks may have to liquidate their long term assets in the secondary market. When some banks take risky investment, they adversely affect the asset price in the secondary market and hence reduce the capability of banks to withstand the liquidity shocks, leading to stronger risk shifting incentives of other banks. Therefore, the risk taking of banks endangers financial stability by rendering the financial system more vulnerable to "panics" on asset market liquidity. In the model setting, a liquidity requirement imposes a tradeoff between improving financial stability in economic downturns and encouraging risk taking in economic booms. Therefore, a counter-cyclical liquidity requirement would be more effective in promoting financial stability.

References

[1] Acharya, V., 2009. “ A theory of systemic risk and design of prudential bank regulation”. *Journal of Financial Stability* 5, 224-255.

[2] Acharya, Viral, Douglas Gale, and Tanju Yorulmazer, 2011. “ Rollover risk and market freezes,” *Journal of Finance* 66, 1177?1209.

[3] Allen, Franklin, and Elena, Carletti, 2008.“The role of liquidity in financial crises,” *2008 Jackson Hole Conference Proceedings, Federal Reserve Bank of Kansas City.*

[4] Bachmann, R., and C. Bayer, 2013. “ Wait-and-See” business cycle?,” *Journal of Monetary Economics*, 60(6), 704 - 719.

[5] Bachmann, R., S. Elstner, and E. R. Sims, 2013. “ Uncertainty and Economic Activity: Evidence from Business Survey Data,” *American Economic Journal: Macroeconomics*, 5(2), 217-249.

[6] Bachmann, R., and G. Moscarini, 2012. “ Business Cycles and Endogenous Uncertainty,” *Working paper.*

[7] Bernanke, Ben S., and Alan Blinder. 1988. “Credit, Money, and Aggregate Demand.? *American Economic Review*, 78(2): 435?39.

[8] Bigio, Saki, 2015. “Endogenous liquidity and the business cycle,” *American Economic Review*, 105, 1883-1927.

[9] Bloom, N., 2009. “The Impact of Uncertainty Shocks,” *Econometrica*, 77(3), 623-685.

[10] Bloom, N., M. Floetotto, I. Saporta-Eksten, and S. J. Terry, 2012. “ Really Uncertain Business Cycles,” *Working paper*, Stanford University and Duke University.

[11] Brunnermeier, Markus K., 2009. “Deciphering the Liquidity and Credit Crunch 2007?2008,” *Journal of Economic Perspective*, 23, 77-100.

[12] Brunnermeier, Markus K., and Lasse Heje Pedersen. 2009. “Market Liquidity and Funding Liquidity.,” *Review of Financial Studies* 22, no.6, 2201?38.

[13] Dell’Ariccia G., L. Laeven, and G. Suarez, 2016. “Bank Leverage and Monetary Policy?s Risk-Taking Channel: Evidence from the United States?, *Journal of Finance*, forthcoming.

[14] Diamond, Douglas W., and Philip H. Dybvig, 1983. “Bank runs, deposit insurance, and liquidity,” *Journal of Political Economy* 91, 401-419.

[15] Diamond, Douglas W., and Raghuram G. Rajan. 2005. "Liquidity Shortage and Banking Crisis.? *Journal of Finance*, 60(2): 615?647

[16] Diamond, Douglas W., and Raghuram G. Rajan, 2012, "Fear of fire sales, illiquidity seeking, and credit freezes" *Quarterly Journal of Economics* 126, 557-591.

[17] Dong, Feng., Jianjun Miao, and Pengfei Wang, 2016. " The perils of credit booms", *working paper.*

[18] Duran, Miguel A., and Ana Lozano-Vivas, 2014, "Risk shifting in the US banking system: An empirical analysis" *Journal of Financial Stability* 13, 64-74.

[19] ECB, March 2016, "Survey on the Access to Finance of Enterprises in the euro area".

[20] Esty, 1996. "A case study of organizational form and risk shifting in the savings and loans industry" *Journal of Financial Economics*, vol 44, pp 57-76.

[21] Fischer, M., C. Haniz, J. Rocholl, and S. Steffen, 2010, "Government Guarantees and Risk Taking Incentives" *Working Paper.*

[22] French, Kenneth R., et al. 2010. *The Squam Lake Report.* Princeton University Press: Princeton, NJ.

[23] Gan, J., 2004. "Banking Market Structure and Financial Stability: Evidence from the Texas Real Estate Crisis in the 1980s" *Journal of Financial Economics*, vol 73, pp 567-601.

[24] Gertler, M., and P. Karadi, 2011. "Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 58(1), 17-34.

[25] Gertler, M., and N. Kiyotaki, 2009. "Financial Intermediation and Credit Policy in Business Cycle Analysis," *Handbook of Monetary Economics.*

[26] Gilchrist, S., J. Sim, and E. Zakrajsek, 2013. "Uncertainty, Financial Frictions and Investment Dynamics," *Working paper.*

[27] He, Zhiguo, and Konstantin Milbradt, 2014. "Endogenous Liquidity and Defaultable Bonds." *Econometrica* 82, 1443-1508.

[28] He, Zhiguo, and Wei Xiong, 2012. "Rollover risk and credit risk." *Journal of Finance* , 67, 391-428.

[29] Holmstrom, Bengt, and Jean Tirole. 1997. "Financial Intermediation, Loanable Funds, and the Real Sector.? *Quarterly Journal of Economics*, 112(1): 35?52.

[30] Ioannidou V., S. Ongena, and J.-L. Peydr09. " Monetary policy and subprime lending: a tall tale of low federal funds rates, hazardous loans and reduced loan spreads?, *European Banking Centre Discussion Paper*, No. 45, 2009.

[31] Jimz G., S. Ongena, J.-L. Peydr J. Saurina, 2014. "Hazardous Times for Monetary Policy: What Do Twenty-Three Million Bank Loans Say About the Effects of Monetary Policy on Credit Risk-Taking??, *Econometrica*, Vol. 82, No. 2, pp. 463-505.

[32] Jensen, M.C. and Meckling, W.H., 1976. "Theory of the firm: Managerial behavior, agency costs and ownership structure" *Journal of Financial Economics* 3, 305-360.

[33] Krishnamurthy, Arvind, 2010. "How debt markets have malfunctioned in the crisis," *Journal of Economic Perspectives* 24, 3?28.

[34] Kurlat, Pablo, 2013. "Lemons Markets and the Transmission of Aggregate Shocks." *American Economic Review* 103, 1463-89.

[35] Landier, Augustin, David Sraer and David Thesmar, 2011. " The risk-shifting hypothesis: Evidence from sub-prime originations." *12th Jacques Polak Annual Research Conference*

[36] Maddaloni A. and J.-L. Peydr14. "Bank Risk-Taking, Securitization, Supervision and Low Interest Rates: Evidence from the Euro Area and U.S. Lending Standards?, *Review of Financial Studies*, Vol. 24, No. 6, pp. 2121-165.

[37] Malherbe, Frederic, 2014. "Self-Fulfilling Liquidity Dry-Ups." *Journal of Finance* 69, 1443-1508.

[38] Martinez-Miera, David, and Rafael Repullo, 2015. "Search for Yield ." *Working Paper*

[39] Morris, Stephen, and Hyun Shin, 2004. "Liquidity black holes." *Review of Finance* 8, 1-18.

[40] Morris, Stephen, and Hyun Shin, 2010. "Illiquidity Component of Credit Risk." *Working Paper.*

[41] Navarro, Gaston, 2014. "Financial Crises and Endogenous Volatility." *Working Paper*

[42] Orlik, A., and L. Veldkamp, 2014. "Understanding Uncertainty Shocks and the Role of Black Swans," *Working paper.*

[43] Rajan R., “Has financial development made the world riskier??, *European Financial Management*, 12(4), 2006, pp. 499?533.

[44] Repullo, Rafael, 2005. “Liquidity, risk taking and the lender of last resort.” *International Journal of Central Banking* 1, 47-79.

[45] Shleifer, Andrei, and Vishny, Robert W. 1992. “Liquidation Values and Debt Capacity: A Market Equilibrium Approach.? *Journal of Finance*, 47(4): 1343?66.

[46] Shleifer, Andrei, and Vishny, Robert W. 2011. “Fire Sales in Finance and Macroeconomics.? *Journal of Economic Perspectives*, 25, 1, pp 29?48.

[47] Stiglitz, J., Weiss, A., 1981. “Credit rationing in markets with imperfect information”. *American Economic Review* 71, 393?410.

[48] Summers, Lawrence H., 2014. “U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound”. *Business Economics* 49, 65?73.

[49] Veldkamp, L.,2005. “Slow Boom, Sudden Crash,” *Journal of Economic Theory*, 124(2), 230-257.

Appendix

Proof of corner solutions With Assumption 1, $U_0(z;\theta)$ reaches its maximum at the corners of θ.

proof The expected payoff of bank

$$\begin{aligned}U_0(z;\theta) =& \frac{1}{2}\lambda \int_0^{x(z,1;\theta)} (1-\frac{x}{p})(1-c\theta)(1+\theta)z - (1-x)R^* dx \\ &+ \frac{1}{2}\lambda \int_0^{x(z,-1;\theta)} (1-\frac{x}{p})(1-c\theta)(1-\theta)z - (1-x)R^* dx \\ &+ \frac{1}{2}(1-\lambda)\max\{(1-c\theta)(1+\theta)z - R^*, 0\} \\ &+ \frac{1}{2}(1-\lambda)\max\{(1-c\theta)(1-\theta)z - R^*, 0\},\end{aligned}$$

where $x(z,s;\theta)$ is the largest liquidity shock that would guarantee positive payoffs at date 2 for banks given s,

$$x(z,s;\theta) = \min\{\frac{z - \frac{R^*}{(1-c\theta)(1+s\theta)}}{\frac{z}{p} - \frac{R^*}{(1-c\theta)(1+s\theta)}}, 1\}.$$

The marginal payoff in θ

$$\begin{aligned}\frac{\partial U_0(z;\theta)}{\partial\theta} =& \frac{1}{2}z(1-c-2c\theta)\left(\lambda \int_0^{x(z,1;\theta)} (1-\frac{x}{p})dx + (1-\lambda)1_{\{}x(z,1;\theta) > 0\}\right) \\ &+ \frac{1}{2}z(-1-c+2c\theta)\left(\lambda \int_0^{x(z,-1;\theta)} (1-\frac{x}{p})dx + (1-\lambda)1_{\{}x(z,-1;\theta) > 0\}\right).\end{aligned}$$

Denote θ^* satisfies that

$$z(1-c\theta)(1-\theta) = R^*.$$

For $\theta > \theta^*$, $x(z,-1;\theta) = 0$. When taking risk $\theta > \theta^*$, the bank will default in negative shocks at date 2. So for $\theta > \theta^*$, the marginal payoff in θ becomes ,

$$\frac{\partial U_0(z;\theta)}{\partial\theta} = \frac{1}{2}z(1-c-2c\theta)\left(\lambda \int_0^{x(z,1;\theta)} (1-\frac{x}{p})dx + (1-\lambda)1_{\{}x(z,1;\theta) > 0\}\right)$$

Given Assumption 1, for all p, the marginal payoff $\frac{\partial U_0(z;\theta)}{\partial\theta}$ is always positive for $\theta > \theta^*$.

Now consider the case $\theta < \theta^*$. For $p > 1$, $x(z,s;\theta) = 1$ for $s = 1$ and -1. The bank will not default as long as $z \geq R^*$.

$$\frac{\partial U_0(z;\theta)}{\partial \theta} = -cz\left(\lambda \int_0^1 (1-\frac{x}{p})dx + (1-\lambda)\right) < 0.$$

For $p \leq 1$, $x(z,s;\theta) < 1$ for $s = 1$ and -1.

$$\begin{aligned}\frac{\partial U_0(z;\theta)}{\partial \theta} =& -cz\lambda \int_0^{x(z,-1;\theta)} (1-\frac{x}{p})dx - cz(1-\lambda) \\ &+ \frac{1}{2}z(1-c-2c\theta)\lambda\left(\int_{x(z,-1;\theta)}^{x(z,1;\theta)} (1-\frac{x}{p})dx\right).\end{aligned}$$

Because $x(z,1;\theta)$ is increasing in θ and $x(z,-1;\theta)$ is decreasing in θ, $\frac{\partial U_0(z;\theta)}{\partial \theta}$ is increasing in θ for c sufficiently small. So there is no local maximum for $\theta < \theta^*$.

As shown previously, $U_0(z;\theta)$ increases for $\theta > \theta^*$.

By continuity of the $U_0(z;\theta)$ in θ, $\theta = \theta^*$ is not a local maximum. For $\theta = 0$, $x(z,-1;0) = x(z,1;1)$,

$$\frac{\partial U_0(z;\theta)}{\partial \theta}|_{\theta=0} = -cz\lambda \int_0^{x(z,-1;\theta)} (1-\frac{x}{p})dx - cz(1-\lambda) < 0.$$

So the optimal solution is either at 0 or 1.

Proof of PROPOSITION 1 (Optimal Choice of Risk) (i) For $p \leq 1$, for $z > R^*$,

$$U_0(z;1) = (1-c)\left(z - \frac{R^*}{2(1-c)}\right)\left(\frac{\lambda}{2}x_H(p,R^*,z) + (1-\lambda)\right)$$

$$U_0(z;0) = (z-R^*)\left(\frac{\lambda}{2}x_L(p,R^*,z) + (1-\lambda)\right).$$

where $x^H(p,R^*,z)$ and $x^L(p,R^*,z)$ are defined respectively as

$$x^H(p,R^*,z) = \frac{z - \frac{R^*}{2(1-c)}}{\frac{z}{p} - \frac{R^*}{2(1-c)}} \tag{43}$$

and

$$x^L(p,R^*,z) = \frac{z-R^*}{\frac{z}{p} - R^*}. \tag{44}$$

Otherwise, for $z \leq R^*$, $U_0(z;1) = U_0(z;0) = 0$.

(ii) For $p > 1$, for $z > R^*$

$$U_0(R^*, z; 1) = (1-c)\left(z - \frac{R^*}{2(1-c)}\right)\left(\frac{\lambda}{2} + (1-\lambda)\right) + (1-c)\frac{\lambda}{2}(1-\frac{1}{p})z$$

and

$$U_0(R^*, z; 0) = (z - R^*)\left(\frac{\lambda}{2} + (1-\lambda)\right) + \frac{\lambda}{2}(1-\frac{1}{p})z.$$

Otherwise, for $z \leq R^*$, $U_0(z; 1) = U_0(z; 0) = 0$.

Solving for z^* (i) For $p \leq 1$, for $z > R^*$, z^* satisfies

$$(1-c)\left(z - \frac{R^*}{2(1-c)}\right)\left(\frac{\lambda}{2}x_H(p, R^*, z) + (1-\lambda)\right) = (z - R^*)\left(\frac{\lambda}{2}x_L(p, R^*, z) + (1-\lambda)\right).$$

Dividing both sides by R^*,

$$(1-c)\left(\alpha - \frac{1}{2(1-c)}\right)\left(\frac{\lambda}{2}x_H(p, 1, \alpha) + (1-\lambda)\right) = (\alpha - 1)\left(\frac{\lambda}{2}x_L(p, 1, \alpha) + (1-\lambda)\right).$$

The equation gives unique solution for α. $z^* = \alpha(p)R^*$.

Proof of the uniqueness of z^* There are two steps to prove the existence of a unique z^*.

Step 1: $z^*/R^* \geq \frac{1}{2c}$.

Note that $\alpha(p)$ is first increasing then decreasing in p for $p < 1$[31], $\alpha(p) \geq min\{\alpha(0), \alpha(1)\} = \frac{1}{2c}$, then $z^* \geq \frac{1}{2c}R^*$.

Step 2: for $z \geq z^* \geq R^*/(2c)$, it can be shown that

$$\frac{\partial U_0(z; 1)}{\partial z} < \frac{\partial U_0(z; 0)}{\partial z}.$$

Note that

$$\frac{\partial U_0(z; 1)}{\partial z} = (1-c)\left(\frac{\lambda}{2}x_H(p, R^*, z) + (1-\lambda)\right) + (1-c)\left(z - \frac{R^*}{2(1-c)}\right)\frac{\lambda}{2}\frac{\partial x_H(p, R^*, z)}{\partial z}.$$

and

$$\frac{\partial U_0(z; 0)}{\partial z} = \left(\frac{\lambda}{2}x_L(p, R^*, z) + (1-\lambda)\right) + (z - R^*)\frac{\lambda}{2}\frac{\partial x_L(p, R^*, z)}{\partial z}.$$

To show $\frac{\partial U_0(z;1)}{\partial z} < \frac{\partial U_0(z;0)}{\partial z}$, we only need to show that holds for $\lambda = 1$. Then we only need to show that $(1-c)x_H(p, R^*, z) < x_L(p, R^*, z)$. It holds for all $z > \frac{R^*}{2c}$.

[31] as shown in Proposition 2

(ii) For $p > 1$, z^* satisfies

$$(1-c)\left(z-\frac{R^*}{2(1-c)}\right)\left(\frac{\lambda}{2}+(1-\lambda)\right)+(1-c)\frac{\lambda}{2}(1-\frac{1}{p})z=(z-R^*)\left(\frac{\lambda}{2}+(1-\lambda)\right)+\frac{\lambda}{2}(1-\frac{1}{p})z.$$

Dividing both sides by R^*,

$$(1-c)\left(\alpha-\frac{1}{2(1-c)}\right)\left(\frac{\lambda}{2}+(1-\lambda)\right)+(1-c)\frac{\lambda}{2}(1-\frac{1}{p})\alpha=(\alpha-1)\left(\frac{\lambda}{2}+(1-\lambda)\right)+\frac{\lambda}{2}(1-\frac{1}{p})\alpha.$$

solving yields,

$$\alpha=\frac{1}{2c}\frac{\frac{1}{2}\lambda+1-\lambda}{(1-\frac{1}{2p})\lambda+1-\lambda}.$$

Then, $z^*=\alpha(p)R^*$.

Proof of Proposition 2: (i) For $p > 1$ it is trivial to prove that $\alpha(p)$ is decreasing in p.

(ii)For $p \leq 1$, given $R^*=1$, $z^*=\alpha(p)$. $\alpha(p)$ is the threshold mean return when banks face no cost of borrowing.

$$\frac{\partial z^*}{\partial p}=-\frac{\frac{\partial U(z^*;1)}{\partial p}-\frac{\partial U(z^*;0)}{\partial p}}{\frac{\partial U(z^*;1)}{\partial z}-\frac{\partial U(z^*;0)}{\partial z}}$$

Because $\frac{\partial U(z;1)}{\partial z}-\frac{\partial U(z;0)}{\partial z}<0$, for all z,

$$\frac{\partial z^*}{\partial p}\left(\frac{\partial U(z^*;1)}{\partial p}-\frac{\partial U(z^*;0)}{\partial p}\right)>0.$$

Note that given R^*,

$$\begin{aligned}\frac{\partial U(z^*;1)}{\partial p}-\frac{\partial U(z^*;0)}{\partial p}=&\frac{\lambda}{2}\left((1-c)(z^*-\frac{R^*}{2(1-c)})\frac{\partial x_H^*}{\partial p}-(z^*-R^*)\frac{\partial x_L^*}{\partial p}\right)\\=&\frac{\lambda}{2}\frac{z}{p^2}\left((1-c)x_H^{*2}-x_L^{*2}\right)\end{aligned}$$

where $x_H^*=x_H(p,R^*,z^*)$ and $x_L^*=x_L(p,R^*,z^*)$.

Because

$$\begin{aligned}\sqrt{1-c}x_H-x_L=&\sqrt{1-c}\frac{z-\frac{R^*}{2(1-c)}}{\frac{z}{p}-\frac{R^*}{2(1-c)}}-\frac{z-R^*}{\frac{z}{p}-R^*}\\=&\frac{\sqrt{1-c}\left(z-\frac{R^*}{2(1-c)}\right)\left(\frac{z}{p}-R^*\right)-(z-R^*)\left(\frac{z}{p}-\frac{R^*}{2(1-c)}\right)}{\left(\frac{z}{p}-\frac{R^*}{2(1-c)}\right)\left(\frac{z}{p}-R^*\right)}\end{aligned}$$

The numerator is a quadratic and concave function in z. For $z > R^*$, there exists a unique $\hat{z}(p)$ that makes the numerator equal to zero. $\hat{z}(p)$ is decreasing in p. At $p = 0$, $z^*(0, R^*) < \hat{z}(p)$ or equivalently $\sqrt{1-c}x_H^* - x_L^* > 0$. So $z^*(p, R^*)$ is increasing. Until p reaches $\tilde{p}$ that makes $z^*(p, R^*) = \hat{z}(p)$, or equivalently $\sqrt{1-c}x_H^* - x_L^* = 0$. After that, for $p > \tilde{p}$, $z^*(p, R^*) > \hat{z}(p)$, or $\sqrt{1-c}x_H^* - x_L^* < 0$. That is, $z^*(p, R^*)$ decreases.

Proof of PROPOSITION 3 (The Existence of R^*) a) At $R^* = \bar{r}$, $R^*\gamma^d(z, p, R^*) = \bar{r}\gamma^d(z, p, 1) \leq \bar{r}$.
b) Given Assumption 2, at $R^* = 2(1-c)z$, the bank takes risk and can only obtain payoffs when receive no early withdrawals $x = 0$. For $p \leq 1$, $R^*\gamma^d(z, p, R^*) = (1-c)z(1-\lambda) > \bar{r}$. For $p > 1$, $R^*\gamma^d(z, p, R^*) \geq (1-c)z > \bar{r}$.
By continuity, there exists R^* satisfy the participation constraint.

Proof of Lemma *$R^*(p, z)$ is decreasing in p.*
Total differentiating the depositors participation constraint:

$$\frac{\partial R^*}{\partial p} = -\frac{R^* \frac{\partial \gamma^d}{\partial p}}{\gamma^d + R^* \frac{\partial \gamma^d}{\partial R}} < 0. \tag{45}$$

Proof of PROPOSITION 5 To prove the dominance of indirect effect for $p < \tilde{p}$, it is enough to show that

$$\frac{\partial \gamma^d / \gamma^d}{\partial p / p} > \frac{\partial \alpha / \alpha}{\partial p / p}. \tag{46}$$

Because $\frac{\partial \gamma^d}{\partial p} \frac{1}{\gamma^d}$ is increasing in p and $\frac{\partial \alpha}{\partial p} \frac{1}{\alpha}$ is decreasing in p, to prove the above inequality, it is enough to show the inequality holds at $p = 0$,

Denote $u_0(\alpha; \theta) = \frac{1}{R^*} U_0(z^*; \theta)$. Total differentiating u_0 with respect to p,

$$\frac{\partial \alpha}{\partial p} = -\frac{\frac{\partial u_0(\alpha;1)}{\partial p} - \frac{\partial u_0(\alpha;0)}{\partial p}}{\frac{\partial u_0(\alpha;1)}{\partial \alpha} - \frac{\partial u_0(\alpha;0)}{\partial \alpha}}$$

As p approaches 0,

$$\frac{\partial \alpha}{\partial p} \frac{1}{\alpha} = \frac{\lambda}{1-\lambda} D(c),$$

with

$$D(c) = 2c \left((1-c)(\frac{1}{2c} - \frac{1}{2(1-c)})^2 - (\frac{1}{2c} - 1)^2 \right).$$

And

$$\frac{\partial \gamma^d}{\partial p} \frac{1}{\gamma^d} = \begin{cases} \frac{\lambda}{1-\lambda} \frac{z - \frac{R^*}{2(1-c)}}{z} & \text{if } z < \alpha R^* \\ \frac{\lambda}{1-\lambda} \frac{z - R^*}{z} & \text{if } z \geq \alpha R^*. \end{cases}$$

With the assumption 3, $(1-c)(1-\lambda)\underline{z} > \frac{1}{1-D(c)}$, the inequality

$$\frac{\partial \alpha}{\partial p}\frac{1}{\alpha} < \frac{\partial \gamma^d}{\partial p}\frac{1}{\gamma^d}$$

holds for all p.

Proof of Proposition 6 The expected payoffs in liquidity shocks with and without risk are expressed below respectively,

$$\begin{aligned} U_0^L(l, R^*, z; 1) =& \frac{1}{2}\int_0^{\psi(p)} \max\left\{y_H(l, R^*, z, x; 1), 0\right\} dx \\ =& \frac{1}{2}\int_0^l (1-l)2(1-c)z + (l-x)\bar{r} - (1-x)R^* \\ &+ \frac{1}{2}\int_l^{x_H} (1-l-\frac{x-l}{p})2(1-c)z - (1-x)R^* \\ =&(1-l)(1-c)zx_H + \frac{l^2}{4}\bar{r} - \frac{(1-c)z}{2p}(x_H-l)^2 - (2-x_H)x_H\frac{R^*}{4} \end{aligned} \tag{47}$$

and

$$\begin{aligned} U_0^L(l, R^*, z; 0) =& \int_0^{\psi(p)} \max\left\{y_H(l, R^*, z, x; 0), 0\right\} dx \\ =& \int_0^l (1-l)z + (l-x)\bar{r} - (1-x)R^* + \int_l^{x_L} (1-l-\frac{x-l}{p})z - (1-x)R^* \\ =&(1-l)zx_L + \frac{l^2}{2}\bar{r} - \frac{z}{2p}(x_L-l)^2 - (2-x_L)x_L\frac{R^*}{2} \end{aligned} \tag{48}$$

Note that $x^H(l, p, R^*, z)$ and $x^L(l, p, R^*, z)$ are defined respectively as

$$x^H(l, p, R^*, z) = \min\{\frac{\frac{\psi(p,l)}{p}z - \frac{R^*}{2c}}{\frac{1}{p}z - \frac{R^*}{2c}}, 1\} \tag{49}$$

and

$$x^L(l, p, R^*, z) = \min\{\frac{\frac{\psi(p,l)}{p}z - R^*}{\frac{1}{p}z - R^*}, 1\}. \tag{50}$$

When the bank is not hit by liquidity shocks, the expected payoffs with and without risk,

$$\begin{aligned} U_0^{NL}(l, R^*, z; 1) =& \frac{1}{2}y_H(l, R^*, z, 0; 1) \\ =&(1-c)\left((1-l)z - \frac{R^*}{2(1-c)}\right) + \frac{1}{2}l\bar{r}, \end{aligned} \tag{51}$$

and

$$U_0^{NL}(l,R^*,z;0) = \frac{1}{2} y_H(l,R^*,z,0;0) \\ = (1-l)z - R^* + l\bar{r}. \tag{52}$$

The expected payoffs are given by

$$U_0(l,R^*,z;\theta) = \lambda U_0^L(l,R^*,z;\theta) + (1-\lambda)U_0^{NL}(l,R^*,z;\theta) \tag{53}$$

where $\theta \in \{0,1\}$.

The threshold mean return is given by $U_0(l,R^*,z;0) = U_0(l,R^*,z;1)$.

(i) For $p > 1$, it is easy to show that

$$z^*(l,p,R^*) = \frac{1}{2c}\frac{1}{1-l}\frac{R^* - \lambda\frac{l^2}{2}\bar{r} - (1-\lambda)l\bar{r}}{1-\frac{\lambda}{2p}(1-l)}.$$

The uniqueness of z^* is easy to show because $\frac{\partial U_0(l,R^*,z;1)}{\partial z} < \frac{\partial U_0(l,R^*,z;0)}{\partial z}$ for all $z > \frac{R^*-l\bar{r}}{1-l}$.

(ii) For $p \leq 1$,

$$\frac{\partial U_0(l,R^*,z;1)}{\partial z} - \frac{\partial U_0(l,R^*,z;0)}{\partial z} = (1-l)(\lambda l + 1 - \lambda)(-c) \\ + \lambda(1-l)^2\left((1-c)x_0^H - x_0^L - \frac{1}{2p}\left((1-c)x_0^{H2} - x_0^{L2}\right)\right) \\ < 0$$

where x_0^H and x_0^L are defined in the basic model. The inequality holds for λ small enough[32]. Then, there is a unique z^* that solves the date 1 problem of the bank.

Proof of Lemma 4 z^* is increasing in l.

Taking derivative of z^* with respect to l,

$$\frac{\partial z^*}{\partial l} = -\frac{\frac{\partial U_0(l,R^*,z^*;1)}{\partial l} - \frac{\partial U_0(l,R^*,z^*;0)}{\partial l}}{\frac{\partial U_0(l,R^*,z^*;1)}{\partial z} - \frac{\partial U_0(l,R^*,z^*;0)}{\partial z}}. \tag{54}$$

Since the denominator is negative (as shown previously),

$$\frac{\partial z^*}{\partial l}\left(\frac{\partial U_0(l,R^*,z^*;1)}{\partial l} - \frac{\partial U_0(l,R^*,z^*;0)}{\partial l}\right) > 0. \tag{55}$$

[32] λ satisfies that $\lambda < \frac{1}{1+\frac{1}{2\tilde{p}}}$, where $\tilde{p}$ is given in the basic model.

For $p \leq 1$. Taking derivative of U_1 with respect to l,

$$\begin{aligned}\frac{\partial U_0(l, R^*, z^*; 1)}{\partial l} =& \left(\lambda x^{H*} + (1-\lambda)\right)(-(1-c)z^*) + (\lambda l + (1-\lambda))\frac{\bar{r}}{2} \\ &+ \lambda\frac{1-c}{p}z^*(1-l)x_0^{H*}\end{aligned}$$

$$\begin{aligned}\frac{\partial U_0(l, R^*, z^*; 0)}{\partial l} =& \left(\lambda x^{L*} + (1-\lambda)\right)(-z^*) + (\lambda l + (1-\lambda))\,\bar{r} \\ &+ \lambda\frac{z^*}{p}(1-l)x_0^{L^*}.\end{aligned}$$

Then the difference

$$\begin{aligned}\frac{\partial U_0(l, R^*, z^*; 1)}{\partial l} - \frac{\partial U_0(l, R^*, z^*; 0)}{\partial l} =& (\lambda l + (1-\lambda))(cz^* - \frac{1}{2}\bar{r}) \\ &+ (\frac{1}{p} - 1)(1-l)\lambda z^*\left((1-c)x_0^{H*} - x_0^{L*}\right)\end{aligned}$$

It is positive when

$$\lambda < \frac{cz^* - \frac{1}{2}\bar{r}}{(1-l)\left((cz^* - \frac{1}{2}\bar{r}) + (\frac{1}{p} - 1)z^*(x_0^{L*} - (1-c)x_0^{H*})\right)} = \lambda^*$$

Because $x_0^L - (1-c)x_0^H < cp$, for

$$\lambda < \frac{cz^* - \frac{1}{2}\bar{r}}{(1-l)\left((cz^* - \frac{1}{2}\bar{r}) + (1-p)cz^*\right)} < \lambda^*,$$

we have

$$\frac{\partial z^*}{\partial l} > 0.$$

For $p > 1$. Taking derivative of U_1 with respect to l,

$$\frac{\partial U_0(l, R^*, z^*; 1)}{\partial l} = -(1-c)z^* + (\lambda l + (1-\lambda))\frac{\bar{r}}{2} + \lambda\frac{1-c}{p}z^*(1-l)$$

$$\frac{\partial U_0(l, R^*, z^*; 1)}{\partial l} = -z^* + (\lambda l + (1-\lambda))\,\bar{r} + \lambda\frac{z^*}{p}(1-l).$$

Then the difference

$$\begin{aligned}\frac{\partial U_0(l, R^*, z^*; 1)}{\partial l} - \frac{\partial U_0(l, R^*, z^*; 0)}{\partial l} =& (\lambda l + (1-\lambda))(cz^* - \frac{1}{2}\bar{r}) + (1 - \frac{1}{p})(1-l)\lambda cz^* \\ >& 0\end{aligned}$$

Immediately, we have

$$\frac{\partial z^*}{\partial l} > 0.$$

Proof of Proposition 7 Optimal Liquidity Holding.

proof

$$\begin{aligned}\frac{\partial}{\partial l}U_0(l,R^*,z;1) =& \lambda\left(-(1-c)zx^H + \frac{1}{2}l\bar{r} + \frac{(1-c)z}{p}(x^H - l)\right)\\ &+ (1-\lambda)(-(1-c)z + \frac{1}{2}\bar{r})\\ \frac{\partial}{\partial l}U_0(l,R^*,z;0) =& \lambda\left(-zx^L + l\bar{r} + \frac{z}{p}(x^L - l)\right) + (1-\lambda)(-z+\bar{r})\end{aligned}$$

(i) For $p > 1$, $x^H = x^L = 1$. Both $\frac{\partial}{\partial l}U_0(l,R^*,z;1)$ and $\frac{\partial}{\partial l}U_0(l,R^*,z;0)$ are negative for all z. So the first derivative for the expected payoff is negative. It is optimal for banks to hold zero liquidity. $l^* = 0$.

(ii) For $p \le 1$, $x^L < x^H < 1$.

$$\begin{aligned}\frac{\partial^2}{\partial l^2}U_1(l,R^*,z;1) =& \lambda\left(-(1-c)z\frac{\partial x^H}{\partial l} + \frac{1}{2}\bar{r} - \frac{(1-c)z}{p}x_0^H\right)\\ =& \lambda\left(-(1-c)z\frac{\partial x^H}{\partial l} + \frac{1}{2}\bar{r} - \frac{(1-c)z}{p}x_0^H\right)\\ =& \lambda\left(-(1-c)z + \frac{1}{2}\bar{r} - (\frac{1}{p} - 1)(1-c)zx_0^H\right)\\ <& 0\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{\partial^2}{\partial l^2}U_0(l,R^*,z;0) =& \lambda\left(-z\frac{\partial x^L}{\partial l} + \bar{r} - \frac{z}{p}x_0^L\right)\\ =& \lambda\left(-z + \bar{r} - (\frac{1}{p} - 1)zx_0^L\right)\\ <& 0\end{aligned}$$

So the first derivative of $U_0(l)$ is decreasing. At $l = 0$,

$$\frac{\partial U_0(l,R^*,z;1)}{\partial l} = \left(\lambda(\frac{1}{p} - 1)x_0^H - (1-\lambda)\right)(1-c)z + (1-\lambda)\frac{1}{2}\bar{r}$$

It is negative when $\lambda < \frac{z - \frac{1}{2(1-c)\bar{r}}}{z - \frac{1}{2(1-c)}\bar{r} + (\frac{1}{p}-1)x_0^H z}$.

$$\frac{\partial U_0(l,R^*,z;0)}{\partial l} = \left(\lambda(\frac{1}{p} - 1)x_0^L - (1-\lambda)\right)z + (1-\lambda)\bar{r}.$$

It is negative when $\lambda < \frac{z-\bar{r}}{z-\bar{r}+(\frac{1}{p}-1)x_0^L z}$.

The optimal liquidity holding can be positive, $l^* > 0$ if λ is large and p is small. Note that when $l = 1$, the bank always defaults. So the optimal liquidity holding is bounded above, $l^* < 1$.

When λ is sufficiently small, $l^* = 0$ for all p.

Proof of Lemma 6 The effect of liquidity regulation on risk-taking z^*:

Taking derivative of z^* with respect to κ,

$$\frac{\partial z^*}{\partial \kappa} = -\frac{\frac{\partial U_0(l(\kappa),R^*,z^*;1)}{\partial \kappa} - \frac{\partial U_0(l(\kappa),R^*,z^*;0)}{\partial \kappa}}{\frac{\partial U_0(l(\kappa),R^*,z^*;1)}{\partial z} - \frac{\partial U_0(l(\kappa),R^*,z^*;0)}{\partial z}}. \tag{56}$$

Similar to the previous proof,

$$\frac{\partial z^*}{\partial \kappa}\left(\frac{\partial U_0(l(\kappa),R^*,z^*;1)}{\partial \kappa} - \frac{\partial U_0(l(\kappa),R^*,z^*;0)}{\partial \kappa}\right) > 0.$$

where

$$\begin{aligned} &\frac{\partial U_0(l(\kappa),R^*,z^*;1)}{\partial \kappa} - \frac{\partial U_0(l(\kappa),R^*,z^*;0)}{\partial \kappa} \\ =&\left(\frac{\partial U_0(l(\kappa),R^*,z^*;1)}{\partial l} - \frac{\partial U_0(l(\kappa),R^*,z^*;0)}{\partial l}\right)\frac{\partial l(\kappa)}{\partial \kappa} \\ &+\left(\frac{\partial U_0(l(\kappa),R^*,z^*;1)}{\partial R^*} - \frac{\partial U_0(l(\kappa),R^*,z^*;0)}{\partial R^*}\right)\frac{\partial R^*}{\partial l}\frac{\partial l(\kappa)}{\partial \kappa}. \end{aligned} \tag{57}$$

With λ sufficiently low, the liquidity requirement always has binding power,

$$\frac{\partial l(\kappa)}{\partial \kappa} = 1.$$

(i) For $p > 1$ There is no defaults induced by liquidity shortage. The illiquidity risk is zero. So the liquid asset holdings do not affect the credit risk, or R^*.

$$\frac{\partial R^*}{\partial l} = 0.$$

So as proved in Lemma 4,

$$\frac{\partial U_0(l(\kappa),R^*,z^*;1)}{\partial \kappa} - \frac{\partial U_0(l(\kappa),R^*,z^*;0)}{\partial \kappa} = \frac{\partial U_0(l(\kappa),R^*,z^*;1)}{\partial l} - \frac{\partial U_0(l(\kappa),R^*,z^*;0)}{\partial l} > 0 \tag{58}$$

Therefore, z^* is increasing in κ.

(ii) For $p \leq 1$ Substituting this into Equation 57,

$$\frac{\partial U_0(l(\kappa), R^*, z^*; 1)}{\partial \kappa} - \frac{\partial U_0(l(\kappa), R^*, z^*; 0)}{\partial \kappa} = \left(\frac{\partial U_0(l(\kappa), R^*, z^*; 1)}{\partial l} - \frac{\partial U_0(l(\kappa), R^*, z^*; 0)}{\partial l}\right) + \left(\frac{\partial U_0(l(\kappa), R^*, z^*; 1)}{\partial R^*} - \frac{\partial U_0(l(\kappa), R^*, z^*; 0)}{\partial R^*}\right)\frac{\partial R^*}{\partial l}. \tag{59}$$

Here I show that the expression is negative by approximating $x_0^H \approx x_0^L \approx p$.

From previous analysis, the first term is given by

$$\begin{aligned} \frac{\partial U_0(l(\kappa), R^*, z^*; 1)}{\partial l} - \frac{\partial U_0(l(\kappa), R^*, z^*; 0)}{\partial l} &= (\lambda\kappa + (1-\lambda))\,(cz^* - \frac{1}{2}\bar{r}) \\ &\quad + \lambda(1-\kappa)z^*(x_0^{L*} - (1-c)x_0^{H*})(1-\frac{1}{p}) \\ &\approx (\lambda\kappa + (1-\lambda))\,(cz^* - \frac{1}{2}\bar{r}) + \lambda(1-\kappa)cz^*(p-1) \end{aligned} \tag{60}$$

The expression is increasing in p.

The second term can be rewritten as

$$\begin{aligned} &\left(\frac{\partial U_0(l(\kappa), R^*, z^*; 1)}{\partial R^*} - \frac{\partial U_0(l(\kappa), R^*, z^*; 0)}{\partial R^*}\right)\frac{\partial R^*}{\partial l} \\ &= \left(\lambda\left(-(1-\frac{1}{2}x_H^*)\frac{1}{2}x_H^* + x_L^*(1-\frac{1}{2}x_L^*)\right) + (1-\lambda)\frac{1}{2}\right)\frac{\partial R^*}{\partial l} \\ &\approx \left(\lambda\frac{p}{2}\left(1-\frac{p}{2}\right) + (1-\lambda)\frac{1}{2}\right)\frac{\partial R^*}{\partial l}. \end{aligned} \tag{61}$$

Because

$$|\frac{\partial R^*}{\partial l}| = \frac{\frac{\partial \gamma^d}{\partial \kappa}\frac{R^*}{\gamma^d}}{1 + \frac{\partial \gamma^d}{\partial R^*}\frac{R^*}{\gamma^d}} \geq \frac{\partial \gamma^d}{\partial l}\frac{R^*}{\gamma^d} \tag{62}$$

For $z < z^*$,

$$\begin{aligned} |\frac{\partial R^*}{\partial l}| &\geq \frac{R^*\frac{\lambda}{2}(1-x_0^{H*})}{\frac{\lambda}{2}(1-x_0^{H*})\kappa + \frac{\lambda}{2}x_0^{H*} + \frac{1-\lambda}{2}} \\ &\approx \frac{R^*\lambda(1-p)}{\lambda(1-p)\kappa + \lambda p + 1 - \lambda}. \end{aligned} \tag{63}$$

For $z \geq z^*$, it gives the same expression.

The second term

$$\begin{aligned} &\left(\frac{\partial U_0(l(\kappa), R^*, z^*; 1)}{\partial R^*} - \frac{\partial U_0(l(\kappa), R^*, z^*; 0)}{\partial R^*}\right)\frac{\partial R^*}{\partial l} \\ &\leq -\left(\lambda\frac{p}{2}\left(1-\frac{p}{2}\right) + (1-\lambda)\frac{1}{2}\right)\frac{R^*\lambda(1-p)}{\lambda(1-p)\kappa + \lambda p + 1 - \lambda}. \end{aligned} \tag{64}$$

So at $p = 0$, the first term becomes

$$\begin{aligned}
&\frac{\partial U_0(l(\kappa), R^*, z^*; 1)}{\partial l} - \frac{\partial U_0(l(\kappa), R^*, z^*; 0)}{\partial l} \\
=&\left(\lambda\kappa + (1-\lambda)\right)(c\frac{1}{2c}R^* - \frac{1}{2}\bar{r}) + \lambda(1-\kappa)c\frac{1}{2c}R^*(p-1) \\
=&\frac{1}{2}\left(\lambda\kappa + (1-\lambda)\right)(R^* - \bar{r}) - \frac{1}{2}\lambda(1-\kappa)R^* \\
=&\frac{1}{2}\left(\frac{2\lambda}{1-\lambda}\kappa + 2 - \lambda\kappa\bar{r} - (1-\lambda)\bar{r}\right) - \frac{\lambda}{1-\lambda}(1-\kappa)
\end{aligned} \tag{65}$$

and it's increasing in κ. The second term becomes,

$$\begin{aligned}
&\left(\frac{\partial U_0(l(\kappa), R^*, z^*; 1)}{\partial R^*} - \frac{\partial U_0(l(\kappa), R^*, z^*; 0)}{\partial R^*}\right)\frac{\partial R^*}{\partial l} \\
=&-\frac{1}{2}(1-\lambda)\frac{R^*\lambda}{\lambda\kappa + 1 - \lambda} \\
=&-\frac{\lambda}{\lambda\kappa + 1 - \lambda}
\end{aligned} \tag{66}$$

and it is decreasing in κ.

For λ satisfies that $(2 - (1-\lambda)\bar{r})(1-\lambda) < 4\lambda$, there exists $\bar{\kappa} \in (0, 1)$, such that for $\kappa < \bar{\kappa}$,

$$\frac{\partial U_0(l(\kappa), R^*, z^*; 1)}{\partial \kappa} - \frac{\partial U_0(l(\kappa), R^*, z^*; 0)}{\partial \kappa} < 0,$$

or z^* is decreasing in κ.